Distributionally Robust Covariance Steering With Optimal Risk Allocation

Joint work with Joshua Pilipovsky & Panagiotis Tsoitras (Georgia Tech)

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Presentation Outline

- **Motivation:** Covariance Steering Problem
  - Polytope State Constraints
  - Convex Cone State Constraints
- **Methodology:** Iterative Risk Allocation
  - Polytope Constraints - Distributionally Robust (DR) Linear Program
  - Convex Cone Constraints - Reverse Union Bound
- Simulation Results
- Conclusion

Covariance Steering for Stochastic Linear Systems

Control of Stochastic Systems

[1] **Papers:**

- A. Hotz & R. E. Skelton, "Covariance control theory", *International Journal of Control*, 1987.
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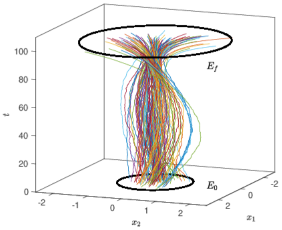

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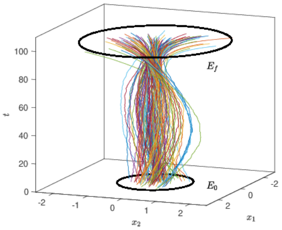

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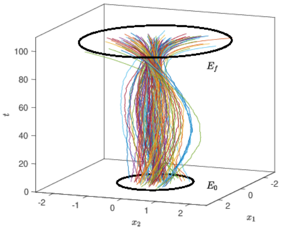

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2. Develop DR iterative risk allocation for both polytopic & convex conic state risk constraints.

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Covariance Steering (CS) Problem

Stochastic LTV Dynamics

$$x_{k+1} = A_k x_k + B_k u_k + D_k w_k, \; k = 0 : N-1$$
$$\mathcal{P}^w = \{\mathbb{P}_w \mid \text{mean} = 0, \text{cov} = \Sigma_w\}$$

Boundary Conditions (BCs) & Cost

$$\mathcal{P}^{x_0} = \{\mathbb{P}_{x_0} \mid \text{mean} = \mu_0, \text{cov} = \Sigma_0\}$$
$$\mathcal{P}^{x_N} = \{\mathbb{P}_{x_N} \mid \text{mean} = \mu_f, \text{cov} = \Sigma_f\}$$
$$J(\mathbf{U}) = \mathbb{E}\left[\mathbf{X}^\top \bar{Q}\mathbf{X} + \mathbf{U}^\top \bar{R}\mathbf{U}\right]$$

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Main Objective

The objective is to steer the trajectories of system in N time steps from $x_0 \sim \mathbb{P}_{x_0} \in \mathcal{P}^{x_0}$ to $x_N \sim \mathbb{P}_{x_N} \in \mathcal{P}^{x_N}$ with $w_k \sim \mathbb{P}_w \in \mathcal{P}^w$.

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Probabilistic State Constraint

$$\mathcal{X} := \{x_k \mid \bigcap_{i=1}^{M} a_i^\top x_k \leq b_i\}.$$

Distributionally Robust (DR) risk constraint.

$$\sup_{\mathbb{P}_\mathbf{X} \in \mathcal{P}^\mathbf{X}} \mathbb{P}_\mathbf{X}\left(\bigwedge_{k=0}^{N} x_k \notin \mathcal{X}\right) \leq \Delta,$$

where $\Delta \in (0, 0.5]$ is the total risk budget.

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CS Problem Statement

$$\min_{\mathbf{U}} \quad J(\mathbf{U})$$
$$\text{s.t.} \quad \text{Dynamics, BCs, DR Risk Constraint.}$$

Propagation of Mean & Covariance

Using the concatenated variables, dynamics can be written as

$$\mathbf{X} = \mathcal{A}x_0 + \mathcal{B}\mathbf{U} + \mathcal{D}\mathbf{W},$$

We adopt the following control policy

$$\mathbf{U} = \mathbf{V} + \mathbf{K}\mathbf{Y}, \quad \text{where} \quad \mathbf{Y} = \mathcal{A}\underbrace{(x_0 - \mu_0)}_{:=y_0} + \mathcal{D}\mathbf{W}$$

$$\implies \bar{\mathbf{Y}} = \mathbb{E}[\mathbf{Y}] = \mathbb{E}[\mathcal{A}y_0 + \mathcal{D}\mathbf{W}] = 0, \quad \text{and}$$

$$\implies \Sigma_{\mathbf{Y}} = \mathcal{A}\Sigma_0\mathcal{A}^\top + \mathcal{D}\Sigma_{\mathbf{W}}\mathcal{D}^\top,$$

Here, the control component $\mathbf{V}$ steers the mean and $\mathbf{K}$ steers the covariance. Then, the mean and covariance of concatenated system state are given by

$$\bar{\mathbf{X}} = \mathcal{A}\mu_0 + \mathcal{B}\mathbf{V},$$

$$\Sigma_{\mathbf{X}} = (I + \mathcal{B}\mathbf{K})\Sigma_{\mathbf{Y}}(I + \mathcal{B}\mathbf{K})^\top$$

$$\implies J(\mathbf{V}, \mathbf{K}) = \underbrace{\bar{\mathbf{X}}^\top \bar{Q}\bar{\mathbf{X}} + \bar{\mathbf{U}}^\top \bar{R}\bar{\mathbf{U}}}_{:=J_\mu} + \underbrace{\mathbf{tr}\left(\bar{Q}\Sigma_{\mathbf{X}} + \bar{R}\Sigma_{\mathbf{U}}\right)}_{:=J_\Sigma}.$$

Relaxing Boundary Conditions

Note that the initial and the terminal state moments can be expressed as follows

$$\mu_0 = E_0 \bar{\mathbf{X}}, \quad \Sigma_0 = E_0 \Sigma_{\mathbf{X}} E_0, \quad \text{and}$$
$$\mu_f = E_N \bar{\mathbf{X}}, \quad \Sigma_f = E_N \Sigma_{\mathbf{X}} E_N.$$

To convexify the problem, we relax the terminal covariance constraint as $\Sigma_f \succeq E_N \Sigma_{\mathbf{X}} E_N$ and subsequently reformulate it as LMI using the Schur complement as

$$\begin{bmatrix} \Sigma_f & E_N (I + \mathcal{B}\mathbf{K}) \Sigma_{\mathbf{Y}}^{\frac{1}{2}} \\ \Sigma_{\mathbf{Y}}^{\frac{1}{2}} (I + \mathcal{B}\mathbf{K})^\top E_N^\top & I \end{bmatrix} \succeq 0.$$

Problems with DR Risk Constraint $\sup_{\mathbb{P}_{\mathbf{X}} \in \mathcal{P}^{\mathbf{X}}} \mathbb{P}_{\mathbf{X}} \left(\bigwedge_{k=0}^{N} x_k \notin \mathcal{X} \right) \leq \Delta$

1. It is a joint DR Risk Constraint
2. It is an infinite dimensional constraint

Prob 1: Joint DR Risk Constraint $\implies$ Individual Risk Constraint

Assume the state constraint $\mathcal{X}$ to be convex polytope & apply Boole's inequality

$$\sup_{\mathbb{P}_{\mathbf{X}} \in \mathcal{P}^{\mathbf{X}}} \mathbb{P}_{\mathbf{X}} \left(\bigwedge_{k=0}^{N} x_k \notin \mathcal{X} \right) \leq \Delta \iff \sup_{\mathbb{P}_{\mathbf{X}} \in \mathcal{P}^{\mathbf{X}}} \mathbb{P}_{\mathbf{X}} \left(\bigwedge_{k=1}^{N} \bigwedge_{i=1}^{M} a_i^\top x_k > b_i \right) \leq \Delta$$

$$\iff \sup_{\mathbb{P}_{x_k} \in \mathcal{P}^{x_k}} \mathbb{P}_{x_k} \left(a_i^\top E_k \mathbf{X} > b_i \right) \leq \delta_{i,k}, \quad \text{and} \quad \sum_{k=1}^{N} \sum_{i=1}^{M} \delta_{i,k} \leq \Delta.$$

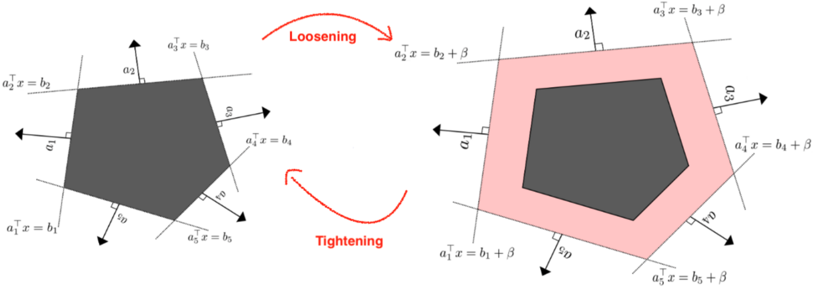


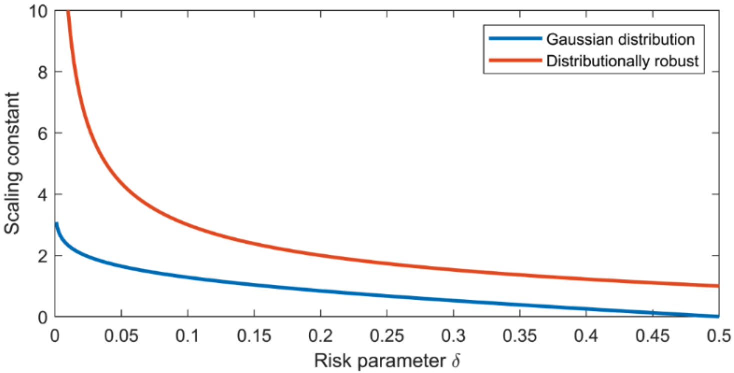
Figure: The original state constraint to the left and the loosened one on the right is shown here.

Probability of Constraint Violation

Gaussian Case

$$\mathbb{P}[x \leq b] = \mathbb{P}[\bar{x} + \sqrt{\Sigma_x} z \leq b] = \mathbb{P}\left[z \leq \frac{b - \bar{x}}{\sqrt{\Sigma_x}}\right] = \Phi\left(\frac{b - \bar{x}}{\sqrt{\Sigma_x}}\right) \leq \delta \iff \bar{x} \geq b - \sqrt{\Sigma_x}\Phi^{-1}(\delta)$$

With Cantelli's Inequality: $\sup_{\mathbb{P}_z} \mathbb{P}_z\left[z \geq \frac{b-\bar{x}}{\sqrt{\Sigma_x}}\right] \leq \frac{1}{1+\left(\frac{b-\bar{x}}{\sqrt{\Sigma_x}}\right)^2} \leq \delta \iff \bar{x} \leq b - \sqrt{\Sigma_x}\sqrt{\frac{1-\delta}{\delta}}$



Prob 2: Deterministic Constraint Tightening (Gaussian vs DR)

The Gaussian Case (when $\mathbb{P}_{\mathbf{X}}$ is Gaussian) Using CDF of Normal Distribution

$$\mathbb{P}_{\mathbf{X}}\left(\bigwedge_{k=1}^{N}\bigwedge_{i=1}^{M} a_i^\top x_k > b_i\right) \leq \Delta \iff a_i^\top \bar{x}_k \leq b_i - \Phi^{-1}(\delta_{i,k}) \left\| \Sigma_{\mathbf{Y}}^{\frac{1}{2}} (I + \mathcal{B}\mathbf{K})^\top E_k^\top a_i \right\|_2$$

The Distributionally Robust Case Using Cantelli's Inequality

$$\sup_{\mathbb{P}_{\mathbf{X}} \in \mathcal{P}^{\mathbf{X}}} \mathbb{P}_{\mathbf{X}}\left(\bigwedge_{k=1}^{N}\bigwedge_{i=1}^{M} a_i^\top x_k > b_i\right) \leq \Delta \iff a_i^\top \bar{x}_k \leq b_i - \underbrace{\sqrt{\frac{1-\delta_{i,k}}{\delta_{i,k}}}}_{:=\mathcal{Q}(1-\delta_{i,k})} \left\| \Sigma_{\mathbf{Y}}^{\frac{1}{2}} (I + \mathcal{B}\mathbf{K})^\top E_k^\top a_i \right\|_2$$

- The tightening constant for DR case is stronger than the Gaussian case for being robust against arbitrary distributions in the set.
- Smaller δ asks for a stricter (greater) tightening.

[1] G.C. Calafiore & L.E. Ghaoui, "On distributionally robust chance constrained linear programs", *Journal of Optimization Theory and Applications*, 130(1), pp.1-22, 2006.

DR Iterative Risk Allocation (DR-IRA)

2-stage optimization framework

- DR-IRA is a 2-stage optimization framework
- The upper stage optimization finds the optimal risk allocation $\delta^\star$
- The lower stage solves the CS problem for the optimal controller $\mathbf{U}^\star$ given the $\delta^\star$

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Lower stage optimization

The value of the objective function after the lower stage optimization for a given risk allocation δ be

$$J^\star(\delta) := \min_{\mathbf{V},\mathbf{K}} J(\mathbf{V},\mathbf{K}).$$

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Upper stage optimization

$$\begin{aligned} \underset{\delta}{\text{minimize}} \quad & J^\star(\delta) \\ \text{subject to} \quad & \sum_{k=1}^{N}\sum_{i=1}^{M} \delta_{i,k} \leq \Delta, \\ & \delta_{i,k} > 0. \end{aligned}$$

DR-IRA Procedure

True Risk $\bar{\delta}_{i,k}$ with $(\mathbf{V}^\star, \mathbf{K}^\star)$

$$\delta_{i,k} \geq \left(1 + \left(\frac{b_i - a_i^\top E_k \bar{\mathbf{X}}^\star}{\left\|\Sigma_{\mathbf{Y}}^{\frac{1}{2}} (I + \mathcal{B}\mathbf{K}^\star)^\top E_k^\top a_i\right\|_2}\right)^2\right)^{-1} =: \bar{\delta}_{i,k}. \quad (1)$$

Note: Constraint i is active if $\delta_{i,k} = \bar{\delta}_{i,k}$, otherwise inactive.

DR-IRA Procedure

1. *Input:* Uniformly allocated risk for all times and constraints defining $\mathcal{X}$.
2. *Output:* $J^\star, \delta^\star, \mathbf{V}^\star, \mathbf{K}^\star$.
3. Loop until cost J converges
   - Break the loop if all or no constraints is active
   - Given current risk, find the optimal control law $\mathbf{V}^\star, \mathbf{K}^\star$ and true risk $\bar{\delta}_{i,k}$.
   - Tighten (reduce the feasible space) all the inactive constraints using $\bar{\delta}_{i,k}$
   - Find the residual risk $\delta_{\text{res}} = \Delta - \sum_k \sum_i \delta_{i,k}$
   - Loosen (increase the feasible space) all the active constraints using δ_{res}.

Convex Cone DR State Constraints

$$\mathcal{X}_c := \left\{ x \in \mathbb{R}^n \mid \|Ax + b\|_2 \leq c^\top x + d \right\}.$$

Relaxing Convex Cone DR State Constraints

Given $\delta_k \in (0, 0.5], \forall k \in [1, N]$, the following DR quadratic risk constraint

$$\sup_{\mathbb{P}_{x_k} \in \mathcal{P}^{x_k}} \mathbb{P}_{x_k} \left[\|Ax_k + b\|_2 \leq c^\top \bar{x}_k + d \right] \geq 1 - \delta_k$$

is a relaxation of the original DR conic risk constraint

$$\sup_{\mathbb{P}_{x_k} \in \mathcal{P}^{x_k}} \mathbb{P}_{x_k} \left[\|Ax_k + b\|_2 \leq c^\top x_k + d \right] \geq 1 - \delta_k.$$

Satisfying DR quadratic constraints

For every time step $k \in [1, N]$, denote $\psi := \|Ax_k + b\|_2$ and $\kappa_k := c^\top \bar{x}_k + d$.

Two sided DR quadratic constraints

The DR quadratic constraint is satisfied if the following constraints are satisfied (subscript k dropped for brevity of notation) for some non-negative $f_1, \ldots, f_n$ and $\beta_1, \ldots, \beta_n$:

$$\sup_{\mathbb{P}_x \in \mathcal{P}^x} \mathbb{P}_x \left[\sum_{i=1}^{n} |\psi_i| \leq f_i \right] \geq 1 - \beta_i \delta, \quad i = [1, N],$$

$$\sum_{i=1}^{n} f_i^2 \leq \kappa^2,$$

$$\sum_{i=1}^{n} \beta_i = 1.$$

Reverse Union Bound Approximation

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Reverse Union Bound (RUB)

Let the events $A_1, \ldots, A_n$ be such that $\mathbb{P}[A_i] \geq \delta_i$ for some $\delta_i \geq 0, \forall i = 1, \ldots, n$. Then,

$$\mathbb{P}\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} \delta_i - (n-1). \tag{2}$$

RUB based approximation of DR quadratic constraints

Let $\epsilon^1_{i,k}, \epsilon^2_{i,k} > 0, \forall i = 1, \ldots, n$ and $k = 1, \ldots, N$. Suppose that the following convex DR SOC constraints hold true for some $\mathbf{V}, \mathbf{K}$, and $\epsilon^1_{i,k} + \epsilon^2_{i,k} \geq 2 - \beta_i \delta_k$:

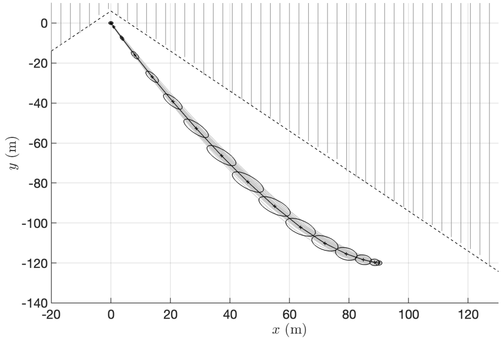
$$a_i^\top E_k \bar{\mathbf{X}} \leq f_{i,k} - b_i - \sqrt{\frac{\epsilon^1_{i,k}}{1-\epsilon^1_{i,k}}} \left\| \Sigma_{\mathbf{Y}}^{\frac{1}{2}} (I + \mathcal{B}\mathbf{K})^\top E_k^\top a_i \right\|_2,$$

$$-a_i^\top E_k \bar{\mathbf{X}} \leq f_{i,k} + b_i - \sqrt{\frac{\epsilon^2_{i,k}}{1-\epsilon^2_{i,k}}} \left\| \Sigma_{\mathbf{Y}}^{\frac{1}{2}} (I + \mathcal{B}\mathbf{K})^\top E_k^\top a_i \right\|_2.$$

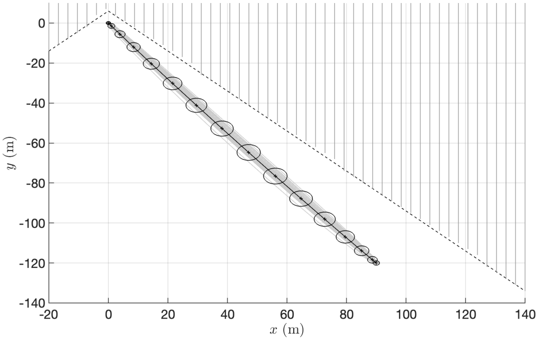
Then the two-sided DR risk constraint holds true as well.

Simulation Results (100 Monte-carlo trials with CS)

- Proximity spacecraft linear model: $\mu_0 = \begin{bmatrix} 90 \\ -120 \end{bmatrix}, \Sigma_0 = 0.1 I_2, \mu_f = 0$, and $\Sigma_f = 0.5\Sigma_0$.
- $w_t, \forall t \in \mathbb{N}$ sampled from multivariate Laplacian distribution.
- Gaussian CS is straight as it exactly knows the probability of constraint violation
- DR CS is curved as it optimizes for worst case probability of constraint violation



DR case with $\Delta = 0.05$.



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Conclusion

Summary

- If distributions of primitive uncertainties (x_0, noises) are non-Gaussian, the state distributions evolve to be non-Gaussian.
- If you ASSUME wrongly everything as Gaussian, it will lead to potentially severe miscalculation of risks.

What next for future?

- Extend the problem for "higher order moment steering" (start with first 4 moments)
- Extend the problem setting to nonlinear systems