LUND UNIVERSITY

The Wheelbot

A Jumping Reaction Wheel Unicycle

Presenter: Zheng Jia

Background & activities

Internship at Intelligent Control Systems (2019-2020)
MPI-IS, *Germany*

⑤

Doctoral student (2021-present)
Automatic Control Lab
Lund University, *Sweden*

B.Eng. in Microelectronics (2009-2011)
Sun Yat-sen University, *China*

①

B.Eng. in Electronic & Information Engineering (2011-2013)
The Hong Kong Polytechnic University, *Hong Kong*

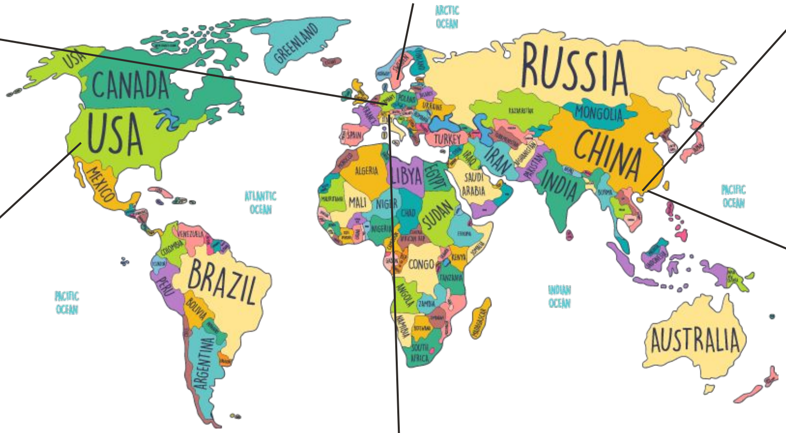
②

M.Sc. in Robotics (2014-2017)
ETH Zürich, *Switzerland*

③

Internship at HiPeRLab (2018)
UC Berkeley, *USA*

④



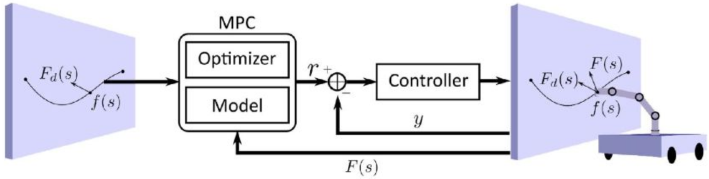
Doctoral project (ELLIIT funded)

Autonomous Force-Aware Swift Motion Control (B14)

PI: Anders Robertsson (LU), co-PI: Lars Nielsen (LiU); with Björn Olofsson (LiU/LU) and Erik Frisk (LiU)

Autonomous Resilient Mobile Robot Path-Tracking Control under Force-Interaction Constraints

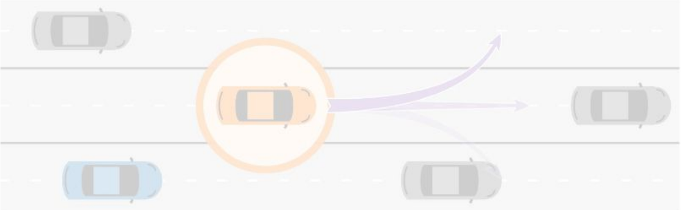
Zheng Jia, Björn Olofsson, Lars Nielsen, Anders Robertsson



- Coordination of vehicle and manipulator motion under force-interaction constraints and path requirement.
- Online (re-)generation of "interaction trajectories" of path, velocity, and force.
- Robust and resilient feedback methods for adjusting the path traversal online (path-velocity scaling).

Resilient Motion Planning and Control for Autonomous Vehicles Using Learning-Based Prediction Techniques

Theodor Westny, Erik Frisk, Björn Olofsson



- For safe planning, a predictive model of surrounding vehicle behavior with a 2–2.5 second prediction horizon is very useful.
- ML-model based on imbalanced real driving data.
- How do models generalize between traffic scenarios?

LINKÖPING UNIVERSITY · LUND UNIVERSITY · LTH FACULTY OF ENGINEERING

WASP WALLENBERG AUTONOMOUS SYSTEMS AND SOFTWARE PROGRAM

WASP associated student (2022)

Thanks again to my supervisors and Karl-Erik.

ELLIIT

Wheelbot (2019-2022)

From 2019-2020 at MPI-IS, *Germany*, under the supervision of Rene and Sebastian, I was working on the wheelbot project.

Wheelbot is a platform for:

1. Learning dynamics with physics knowledge [1]
2. Nonlinear system control
3. Distributed control
4. Education (candidate for our control courses)

Andreas Rene Geist

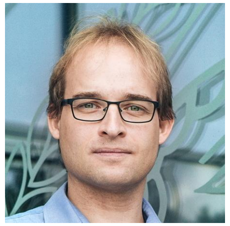
Doctoral student at
MPI-IS, *Germany*

Jonathan Fiene

Robotics leader
MPI-IS, *Germany*

Naomi Tashiro

Researcher
MPI-IS, *Germany*



Sebastian Trimpe

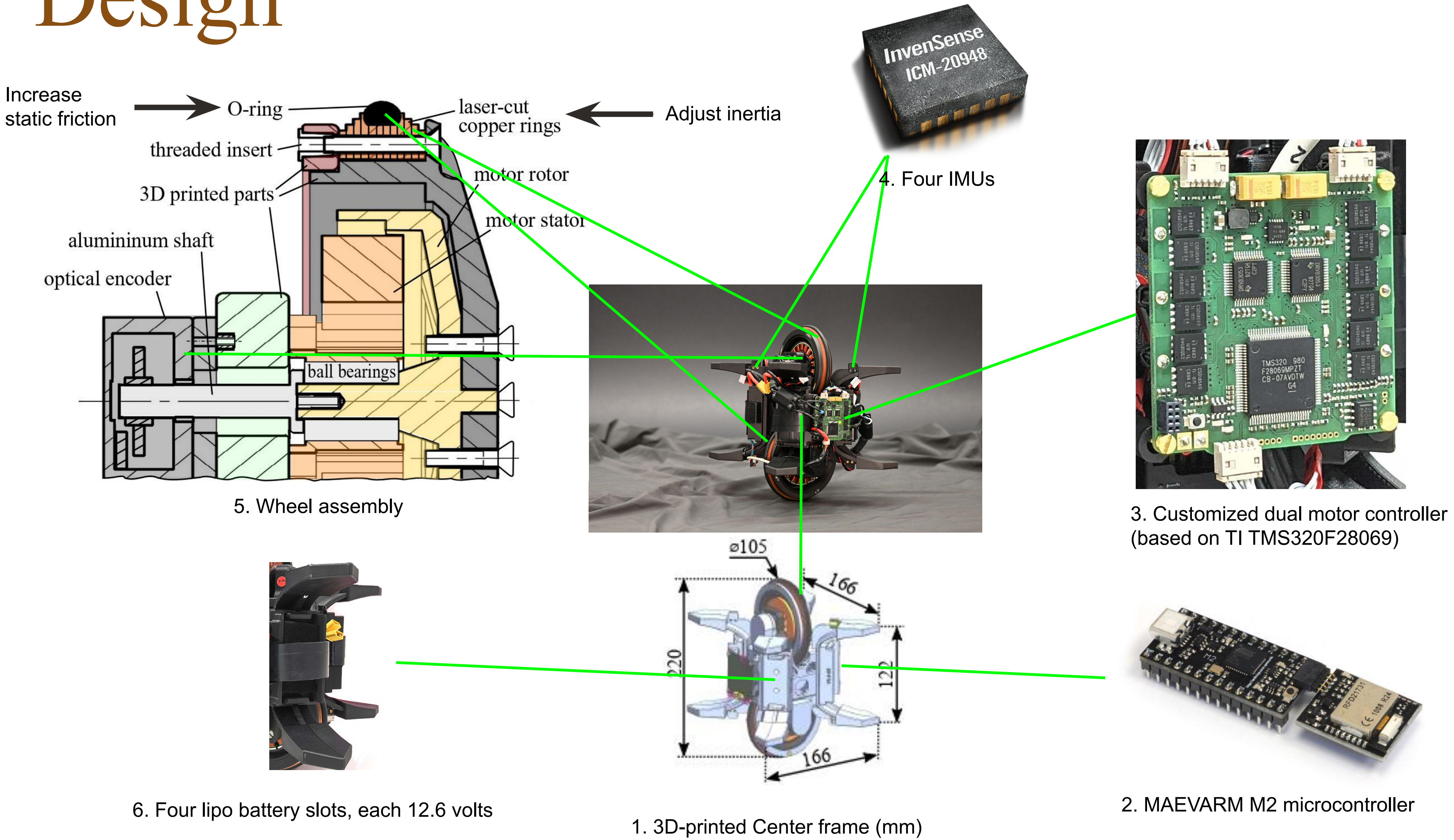
Professor
RWTH, *Germany* since 2020

1. Rath, Lucas, Andreas René Geist, and Sebastian Trimpe. "Using Physics Knowledge for Learning Rigid-body Forward Dynamics with Gaussian Process Force Priors." In *Conference on Robot Learning*, pp. 101-111. PMLR, 2022.

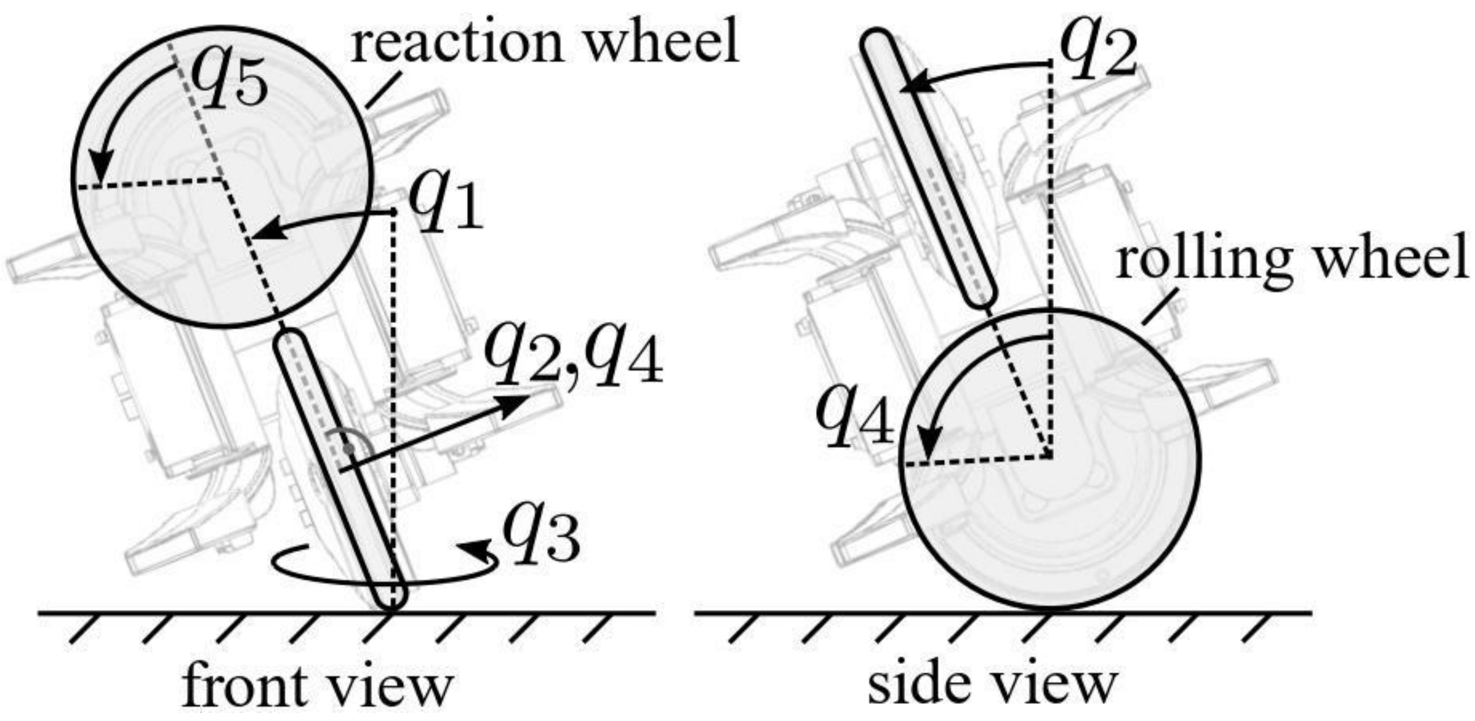
Features

- Off-the-shelf components + 3D printed parts: easy to build
- Symmetric: jump onto one wheel from any initial conditions
- Non-holonomic and underactuated
- Two coupled unstable degrees of freedom
- Nonlinear system

Design


Increase static friction
O-ring
laser-cut copper rings
Adjust inertia
threaded insert
3D printed parts
motor rotor
motor stator
alumininum shaft
optical encoder
ball bearings
5. Wheel assembly
InvenSense ICM-20948
4. Four IMUs
3. Customized dual motor controller
(based on TI TMS320F28069)
ø105
166
220
122
166
6. Four lipo battery slots, each 12.6 volts
1. 3D-printed Center frame (mm)
2. MAEVARM M2 microcontroller

Modeling



q_1 : roll

q_2 : pitch

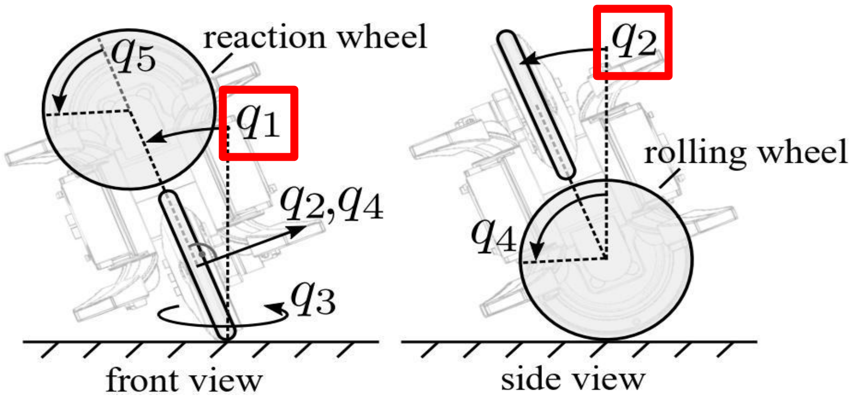
q_3 : yaw

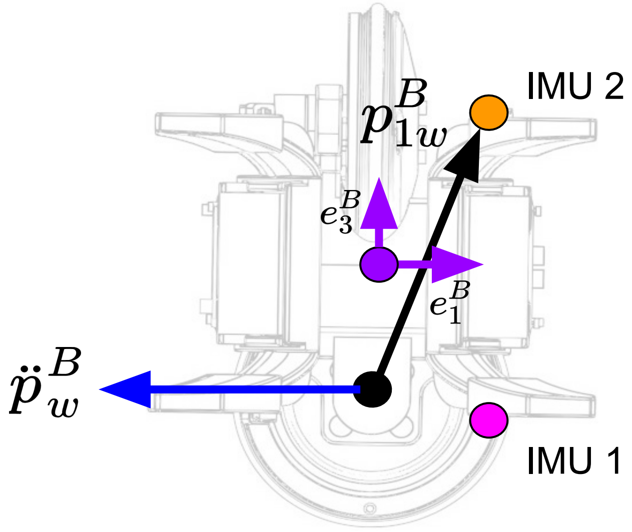
q_4 : rolling wheel

q_5 : upper wheel

Tilt estimation

1. IMUs $\Longrightarrow g^B \Longrightarrow$ Roll & Pitch q_1, q_2





1. Trimpe, Sebastian, and Raffaello D'Andrea. "Accelerometer-based tilt estimation of a rigid body with only rotational degrees of freedom." In *2010 IEEE International Conference on Robotics and Automation*, pp. 2630-2636. IEEE, 2010.
2. Muehlebach, Michael, and Raffaello D’Andrea. "Accelerometer-based tilt determination for rigid bodies with a nonaccelerated pivot point." *IEEE Transactions on Control Systems Technology* 26, no. 6 (2017): 2106-2120.

Tilt estimation

1. IMUs $\Longrightarrow g^B \Longrightarrow$ Roll & Pitch q_1, q_2
2. Measurement of i_{th} accelerometer

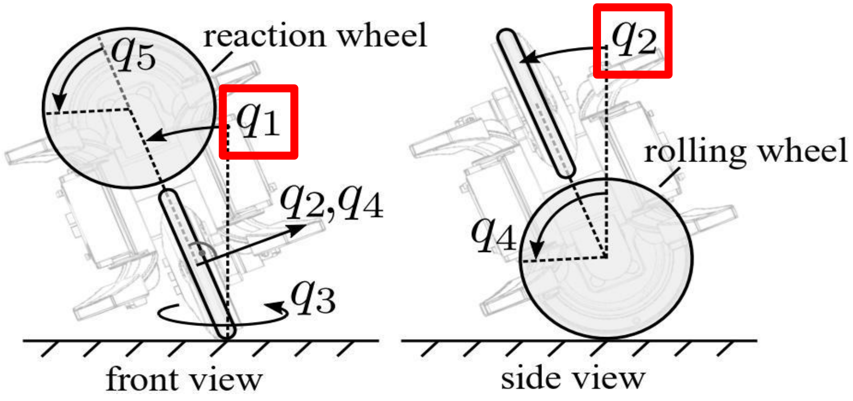
   □ (red): unknowns

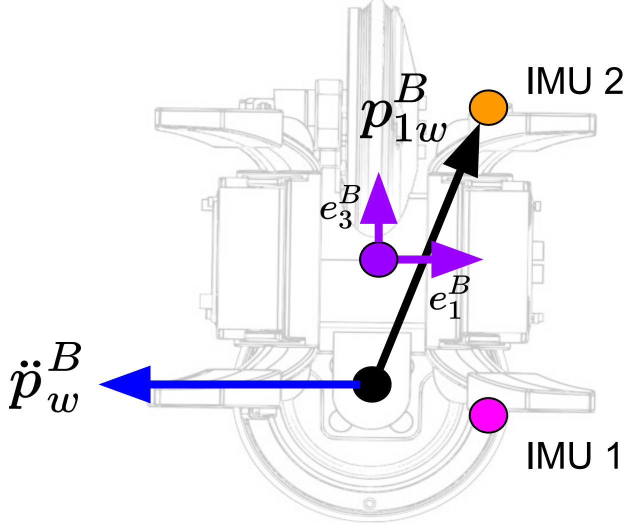
   □ (blue): calculated

   □ (black): knowns

$$m_i^B = \ddot{p}_w^B + \Omega^B p_{iw}^B - g^B + n_i^B \text{ and } \Omega^B := \Omega^B(\omega, \dot{\omega})$$

$$\Longrightarrow M = QP + N, \text{ and } Q = [g^B \quad \Omega^B] \in \mathbb{R}^{3\times 4}$$





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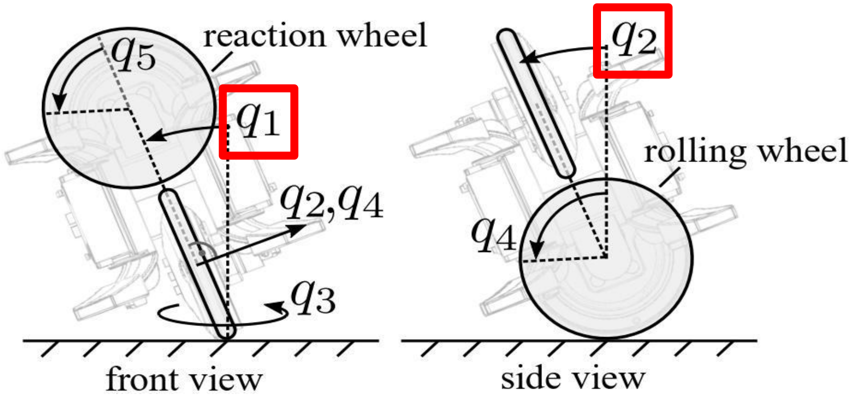
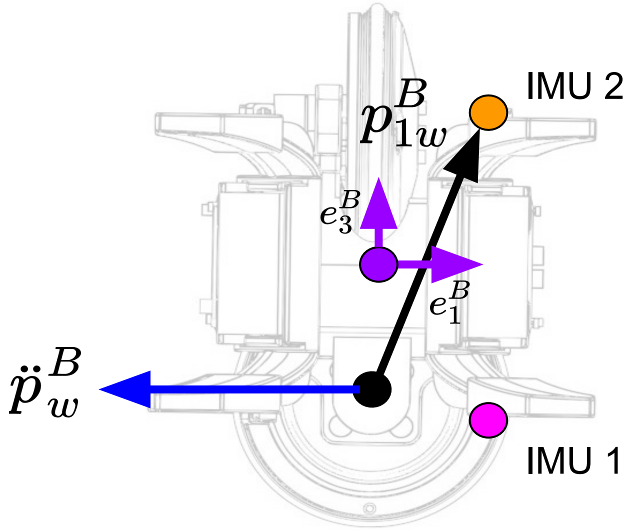
3. $$\min_{\hat{Q}} \mathbb{E}\left[\| \hat{Q} - Q \|_F^2\right] \text{ subj. to } \mathbb{E}\left[\hat{Q}\right] = Q$$

$$\Longrightarrow \hat{Q} = \left[\hat{g}^B, \hat{\Omega}^B\right] = M[X_1^\star, X_2^\star]$$





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$$\Longrightarrow \hat{Q} = \left[\hat{g}^B, \hat{\Omega}^B\right] = M[X_1^\star, X_2^\star]$$

$$\Updownarrow$$

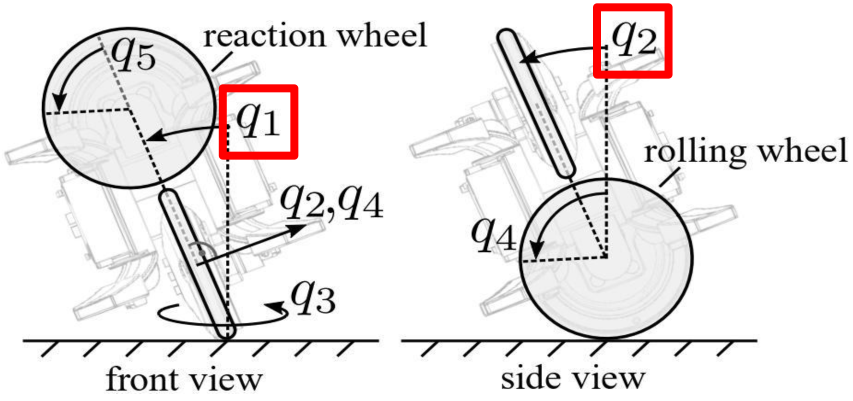
4. $$\min_{X^\star} \mathbb{E}[\| MX^\star - Q \|_F^2] \text{ subj. to } \mathbb{E}[MX^\star] = Q$$

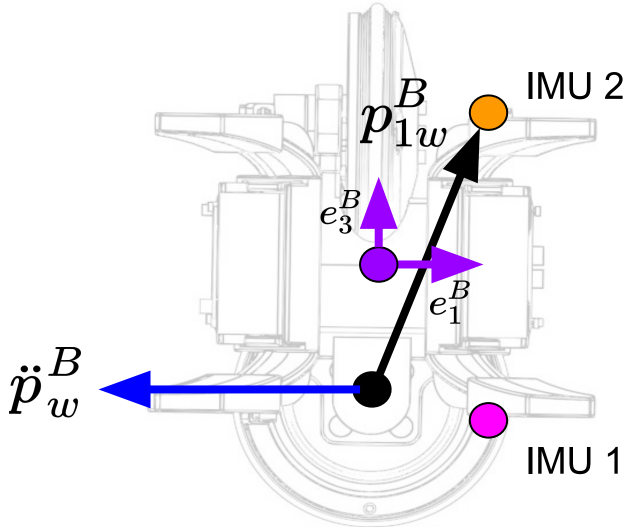
$$\Longrightarrow \hat{g}^B = MX_1^\star \qquad X_1^\star : \text{purely dependent on } p_{iw}^B$$

$$\Longrightarrow \hat{q}_1, \hat{q}_2$$

$$\Longrightarrow R(\hat{q}_1), R(\hat{q}_2)$$

5. Fusion with gyro measurements $\dot{\hat{q}}_1 = R(\hat{q}_2)\omega_{gyro}^B$: complementary filter





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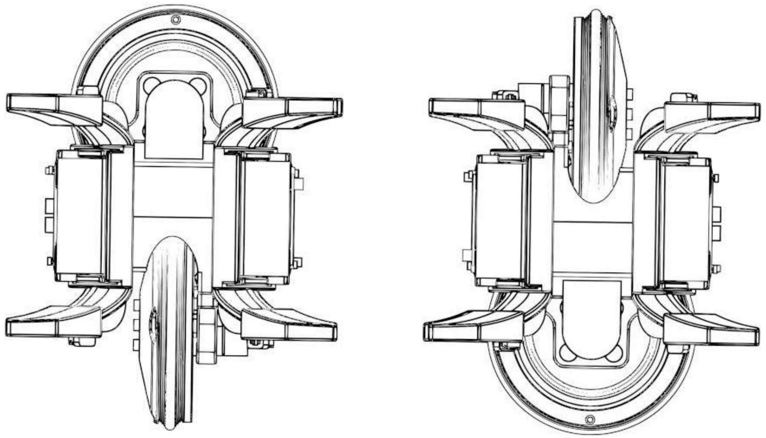
Linearized dynamics



$$q = 0, \dot{q} = 0$$

	q_1	$\dot{q}_1$	q_2	$\dot{q}_2$	q_3	$\dot{q}_3$	q_4	$\dot{q}_4$	q_5	$\dot{q}_5$	u_1	u_2
q_1	0	1	0	0	0	0	0	0	0	0	0	0
$\dot{q}_1$	73.1	0	0	0	0	0	0	0	0	0	0	-51.11
q_2	0	0	0	1	0	0	0	0	0	0	0	0
$\dot{q}_2$	0	0	167	0	0	0	0	0	0	0	-439.3	0
q_3	0	0	0	0	0	1	0	0	0	0	0	0
$\dot{q}_3$	0	0	0	0	0	0	0	0	0	0	0	0
q_4	0	0	0	0	0	0	0	1	0	0	0	0
$\dot{q}_4$	0	0	-153	0	0	0	0	0	0	0	635.3	0
q_5	0	0	0	0	0	0	0	0	0	1	0	0
$\dot{q}_5$	-73.1	0	0	0	0	0	0	0	0	0	0	2039
	roll		pitch		yaw		rolling wheel		upper wheel		rolling wheel	upper wheel

A (columns q_1 to $\dot{q}_5$) $\quad$ B (columns u_1, u_2)

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_5 \\ \dot{q}_5 \\ q_2 \\ \dot{q}_2 \\ q_4 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_5 \\ \dot{q}_5 \\ q_2 \\ \dot{q}_2 \\ q_4 \\ \dot{q}_4 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\text{roll, pitch, upper and rolling wheels dynamics}}$$

controllable

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_3 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \dot{q}_3 \\ 0 \end{bmatrix}}_{\text{yaw dynamics}}$$

uncontrollable

LQR for stabilization

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_5 \\ \dot{q}_5 \\ q_2 \\ \dot{q}_2 \\ q_4 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_5 \\ \dot{q}_5 \\ q_2 \\ \dot{q}_2 \\ q_4 \\ \dot{q}_4 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\text{roll, pitch, upper and rolling wheels dynamics}}$$

controllable

sample time: 0.01s

$Q, R:$ diagonal

$$u = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} \hat{q}_1 - \bar{q}_1 & \dot{\hat{q}}_{1,\text{gyro}} & q_{5,\text{encoder}} & \dot{\hat{q}}_{5,\text{encoder}} \\ \hat{q}_2 - \bar{q}_2 & \dot{\hat{q}}_{2,\text{gyro}} & q_{4,\text{encoder}} & \dot{\hat{q}}_{4,\text{encoder}} \end{bmatrix}$$

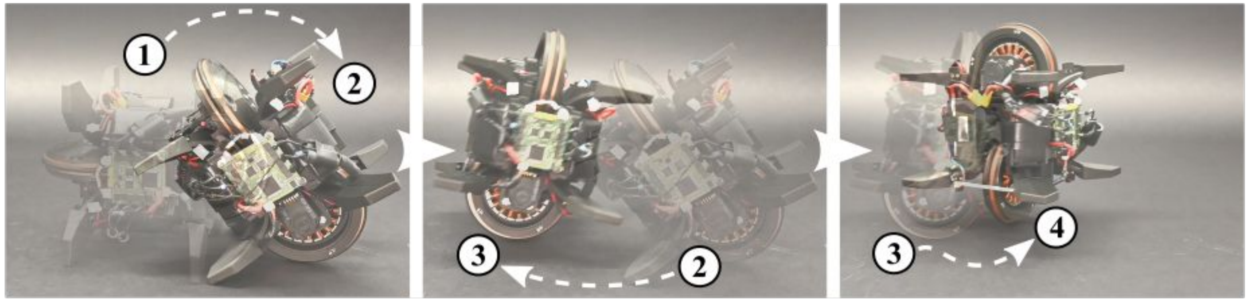
$\bar{q}_1, \bar{q}_2$: bias in tilt estimator

$\dot{\hat{q}}_1, \dot{\hat{q}}_2$: gryo measurements

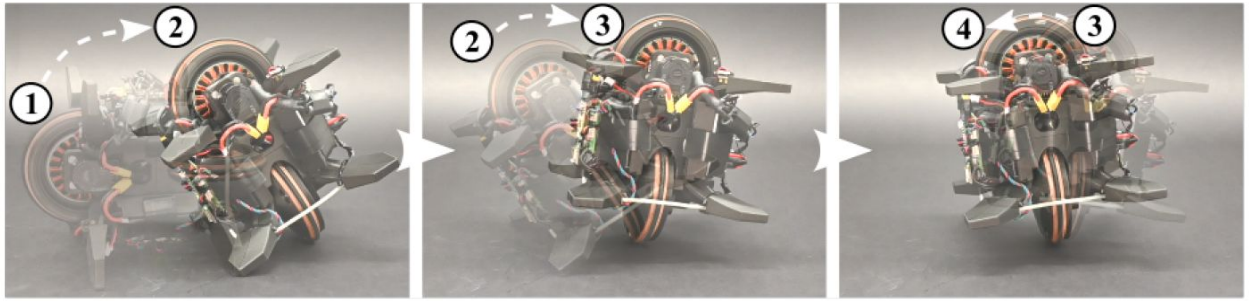
$q_{4,\text{encoder}}, \dot{q}_{4,\text{encoder}}, q_{5,\text{encoder}}, \dot{q}_{5,\text{encoder}}$: measurement from optical encoders

Self-elevation

roll-up



stand-up



Both elevations are based on engineered feed-forward control
Could be a testbed for learning-based control for erection

Can we control yaw?

WARNING

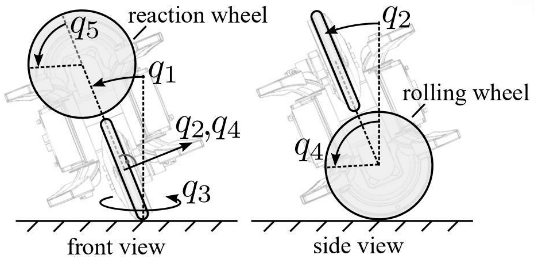
$q(0) = q_l$, with $q_1(0) \neq 0, q_2(0) \neq 0$

1. Apply LQR control law $u_0 = K_{lqr} * q_0(t)$
$\Longrightarrow \dot{q}_0(t) = f(q_0(t), u_0(t))$
$\Longrightarrow$ nominal stabilization trajectory $(q_0(t), u_0(t))$ with initial conditions (q_l, u_l)

2. Linearize about $(q_0(t), u_0(t))$

$$\Delta\dot{q} = A(q_0(t), u_0(t))\Delta q + B(q_0(t), u_0(t))\Delta u$$

But only $A(q_l, u_l), B(q_l, u_l)$ are known. Look at $A(q_l, u_l), B(q_l, u_l)$ at t=0



$q_1(0) \neq 0, q_2(0) \neq 0$

	q_1	$\dot{q}_1$	q_2	$\dot{q}_2$	q_3	$\dot{q}_3$	q_4	$\dot{q}_4$	q_5	$\dot{q}_5$	u_1	u_2
q_1	0	1	0	0	0	0	0	0	0	0	0	0
$\dot{q}_1$	63.31	0	0	0	0	0	0	0	0	0	0	−51.11
q_2	0	0	0	1	0	0	0	0	0	0	0	0
$\dot{q}_2$	0	0	146.3	0.24	0	−0.93	0	0.07	0	0	−439.3	0
q_3	0	0	0	0	0	1	0	0	0	0	0	0
$\dot{q}_3$	0	0	−4.58	−0.48	0	0.29	0	−0.14	0	0	0	0
q_4	0	0	0	0	0	0	0	1	0	0	0	0
$\dot{q}_4$	0	0	−130	0.24	0	−1.14	0	0.07	0	0	635.3	0
q_5	0	0	0	0	0	0	0	0	0	1	0	0
$\dot{q}_5$	−63.31	0	0	0	0	0	0	0	0	0	0	2039
	roll		pitch		yaw		rolling wheel		upper wheel		rolling wheel	upper wheel

$A(q_l, u_l)$ $\quad$ $B(x_l, u_l)$

$$\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} = A_{9x9}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} + B_{9x9}u$$

dynamics excluding yaw angle q_3

controllable

$$\frac{d}{dt}q_3 = \dot{q}_3$$

yaw angle dynamics

uncontrollable

$\Longrightarrow$ yaw rate $\dot{q}_3$ controllable

How difficult to control?

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} = A_{9x9}(t)\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} + B_{9x9}(t)u}_{\text{dynamics excluding yaw angle } q_3}$$

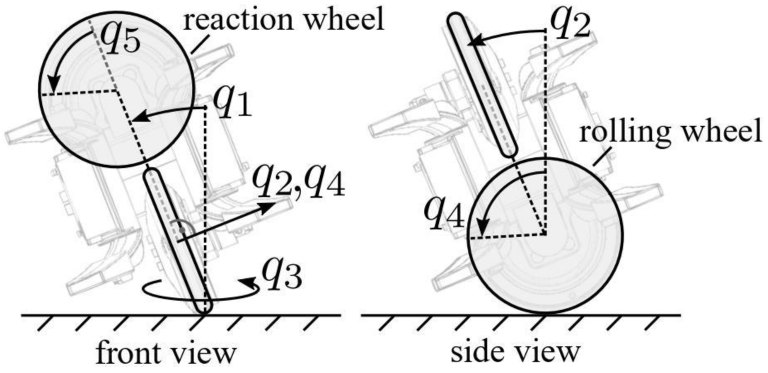
controllable

$$x = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \text{ excluding } q_3, q_4, q_5, \dot{q}_5$$

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = A_{\text{red}}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} + B_{\text{red}}u}_{\text{roll, pitch yaw rate, upperwheel rate dynamics}}$$

controllable

t=0



How difficult to control?

$$\frac{d}{dt}\underbrace{\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} = A_{9x9}(t)\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} + B_{9x9}(t)u}_{\text{dynamics excluding yaw angle } q_3}$$

controllable

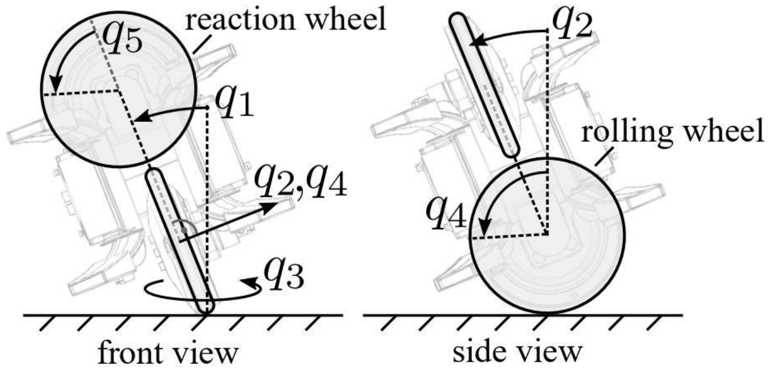
$$\Delta\dot{x}(t) = A_{\text{red}}(t)\Delta x + B_{\text{red}}(t)\Delta u$$

$$x = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \text{ excluding } q_3, q_4, q_5, \dot{q}_5$$

$$\frac{d}{dt}\underbrace{\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = A_{\text{red}}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} + B_{\text{red}}u}_{\text{roll, pitch yaw rate, upperwheel rate dynamics}}$$

controllable

t=0



How difficult to control?

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} = A_{9x9}(t)\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} + B_{9x9}(t)u}_{\text{dynamics excluding yaw angle } q_3}$$

controllable

$$x = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \text{ excluding } q_3, q_4, q_5, \dot{q}_5$$

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = A_{\text{red}}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} + B_{\text{red}}u}_{\text{roll, pitch yaw rate, upperwheel rate dynamics}}$$

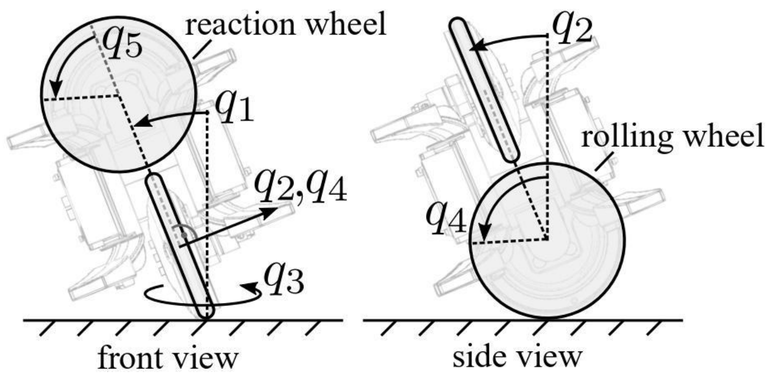
controllable

t=0

$$\Delta\dot{x}(t) = A_{\text{red}}(t)\Delta x + B_{\text{red}}(t)\Delta u$$

For a short amount of time with $t_0 = 0,\ t_f = 0.1$ sec

$$\Delta\dot{x}(t) \approx A_{\text{red}}\Delta x + B_{\text{red}}\Delta u$$



How difficult to control?

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} = A_{9\text{x}9}(t)\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ q_4 \\ \dot{q}_4 \\ q_5 \\ \dot{q}_5 \end{bmatrix} + B_{9\text{x}9}(t)u}_{\text{dynamics excluding yaw angle } q_3}$$

controllable

$$x = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \text{ excluding } q_3, q_4, q_5, \dot{q}_5$$

$$\underbrace{\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = A_{\text{red}}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} + B_{\text{red}}u}_{\text{roll, pitch yaw rate, upperwheel rate dynamics}}$$

controllable

t=0

$$\Delta\dot{x}(t) = A_{\text{red}}(t)\Delta x + B_{\text{red}}(t)\Delta u$$

For a short amount of time with $t_0 = 0,\ t_f = 0.1$ sec

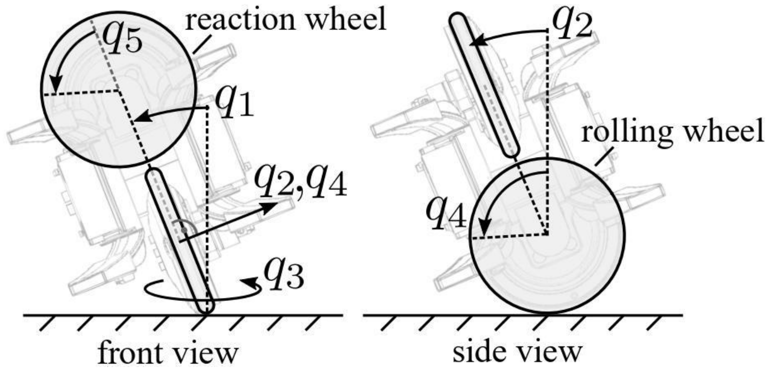
$$\Delta\dot{x}(t) \approx A_{\text{red}}\Delta x + B_{\text{red}}\Delta u$$

Yaw rate deviation $\Delta x_5(0) = 1\,\text{rad/s}$

$$\Delta x(0) = [0, 0, 0, 0, 1, 0]^T$$

Energy driving $\Delta x \Rightarrow 0$

$$E = \Delta x(0)^T \underbrace{W_c^{-1}(x_l, u_l, 0, t_f)}_{\text{finite time ctrl. Gramian [1]}} \Delta x(0)$$



1. Van Loan, Charles. "Computing integrals involving the matrix exponential." *IEEE transactions on automatic control* 23, no. 3 (1978): 395-404.

How difficult to control?

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dynamics excluding yaw angle q_3

controllable

$$x = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \text{ excluding } q_3, q_4, q_5, \dot{q}_5$$

$$\frac{d}{dt}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = A_{\text{red}}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} + B_{\text{red}}u$$

roll, pitch yaw rate, upperwheel rate dynamics

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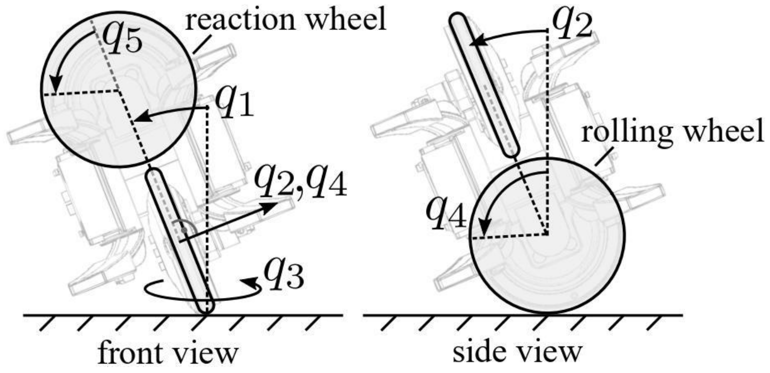

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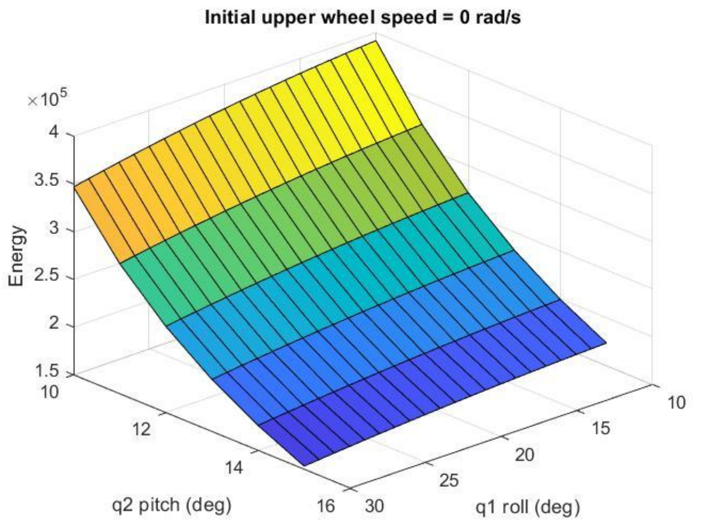
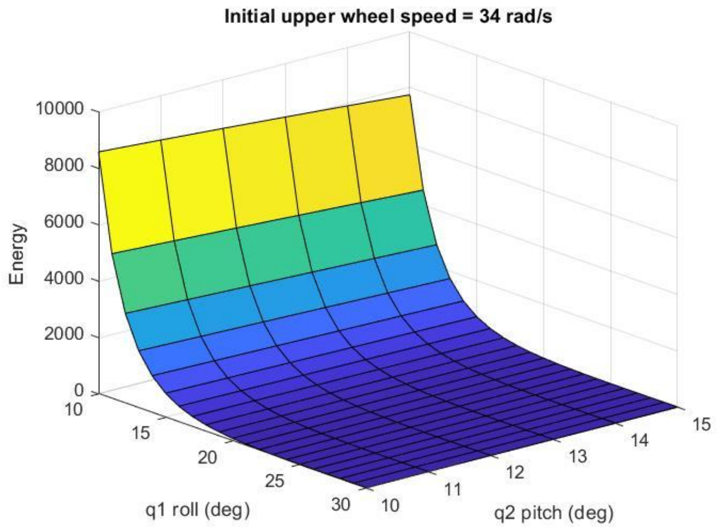
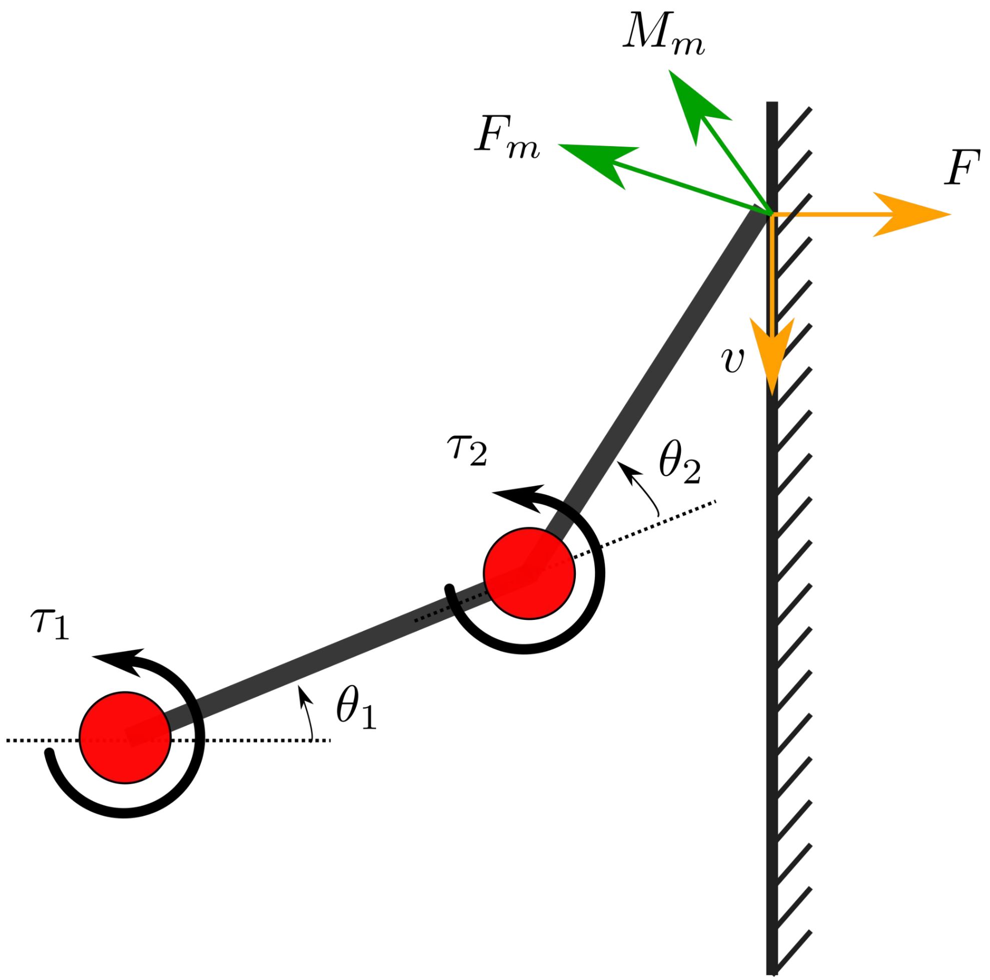


$\Longrightarrow$ easier to control with larger PITCH angle



$\Longrightarrow$ easier to control with larger ROLL angle

Doctoral project (ELLIIT funded)



Given force-torque feedback at the end-effector F_m and M_m, control the normal force and motion along the surface

$$F, v$$

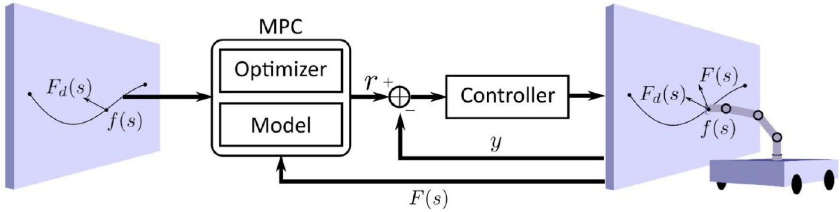
by controlling τ_1, τ_2

What we are looking for is an

optimization framework considering both interaction force and path

Autonomous Resilient Mobile Robot Path-Tracking Control under Force-Interaction Constraints

Zheng Jia, Björn Olofsson, Lars Nielsen, Anders Robertsson



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