Combined Path Velocity Control and Impedance Control for Path-Tracking on Surfaces

Z. Jia[1] B. Olofsson[1,2] L. Nielsen[2] A. Robertsson[1]

[1]Department of Automatic Control
Lund University

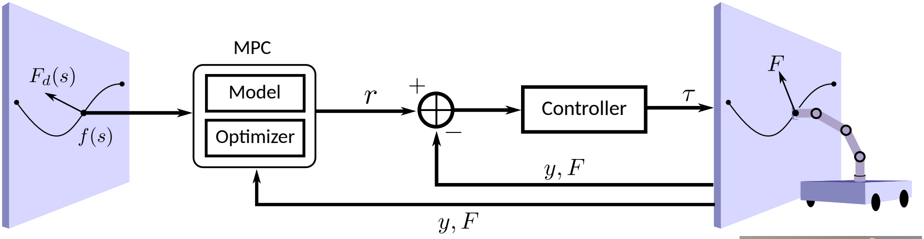
[2]Division of Vehicular Systems, Department of Electrical Engineering
Linköping University

Friday Seminar, Feb 2023

Table of Contents

Table of Contents

Problem description




Heron

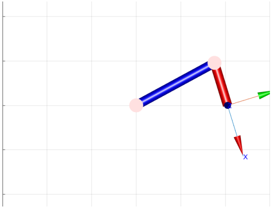
- Mobile manipulator
- Force-torque sensor
- Motion planning & coordination, e.g., model predictive control (MPC)[1]
- Feedback control loops
- Manipulation experiments on the Heron platform

[1] M. Mahdi Ghazaei Ardakani et al. "Model Predictive Control for Real-Time Point-to-Point Trajectory Generation". In: *IEEE Transactions on Automation Science and Engineering* 2 (2019), pp. 972–983

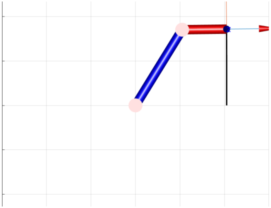
Table of Contents

Time-optimal path-tracking for manipulator

Find nominal time-optimal joint motion to follow a predefined geometric path.



(a) Initial pose



(b) Final pose and path

Time-optimal path-tracking for manipulator

Manipulator dynamics in $q \in \mathcal{R}^n$

Joint angles q can be written as a function of the applied joint torques τ as

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F_s(q)\mathrm{sgn}(\dot{q}) + G(q) \in \mathcal{R}^n \tag{1}$$

Joint path $q(s)$

A joint path $q(s)$ is a function of a scalar path coordinate $s(t) \in \mathcal{R}$. W.l.o.g., $s(0) = 0 \leq s(t) \leq 1 = s(T)$. By chain rule,

$$\dot{q}(s) = q'(s)\dot{s} \tag{2}$$

$$\ddot{q}(s) = q'(s)\ddot{s} + q''(s)\dot{s}^2 \tag{3}$$

Manipulator dynamics in s

Manipulator dynamics in $q \in \mathcal{R}^n$ becomes dynamics in $s \in \mathcal{R}$.

$$\tau(s) = m(s)\ddot{s} + c(s)\dot{s}^2 + g(s) \tag{4}$$

Time-optimal path tracking problem

Time-optimal path tracking problem [1]

$$
\begin{aligned}
\underset{T,s(\cdot),\tau(\cdot)}{\text{minimize}} \quad & T \\
\text{subject to} \quad & \tau(s) = m(s)\ddot{s} + c(s)\dot{s}^2 + g(s) \\
& s(0) = 0 \\
& s(T) = 1 \\
& \dot{s}(0) = \dot{s}_0 \\
& \dot{s}(T) = \dot{s}_T \\
& \dot{s}(t) \geq 0 \\
& \bar{\tau}(s(t)) \geq \tau(t) \geq \underline{\tau}(s(t))
\end{aligned}
$$

Question to people in the study circle of convex optimization

Is this a convex problem?

[1] Diederik Verscheure et al. "Time-optimal path tracking for robots: A convex optimization approach". In: *IEEE Transactions on Automatic Control* 54.10 (2009), pp. 2318–2327

Change of variables

Change of variables and additional constraints

Assume $\dot{s}(t) \geq 0$ and $\dot{s}(t) > 0$ almost everywhere, and introduce

$$a(s) = \ddot{s} \tag{5}$$

$$b(s) = \dot{s}^2 \tag{6}$$

Then $b'(s) = 2a(s)$ is implied since

$$\dot{b}(s) = b'(s)\dot{s} \tag{7}$$

$$\dot{b}(s) = \frac{d(\dot{s}^2)}{dt} = 2\ddot{s}\dot{s} = 2a(s)\dot{s} \tag{8}$$

Change of integration variables

$$T = \int_0^T 1dt = \int_{s(0)}^{s(T)} \frac{1}{\dot{s}} ds = \int_0^1 \frac{1}{\dot{s}} ds = \int_0^1 \frac{1}{\sqrt{b(s)}} ds \tag{9}$$

Reformulation

Change of variables: $a(s) = \ddot{s}$, $b(s) = \dot{s}^2$, $b'(s) = 2a(s)$, $T = \int_0^1 b(s)^{-1/2} ds$

Time-optimal path tracking problem[1] (function in t)

$$\underset{T, s(\cdot), \tau(\cdot)}{\text{minimize}} \quad T$$

$$\tau(t) = m(s(t))\ddot{s} + c(s(t))\dot{s}^2 + g(s(t))$$
$$s(0) = 0$$
$$s(T) = 1$$
$$\dot{s}(0) = \dot{s}_0$$
$$\dot{s}(T) = \dot{s}_T$$
$$\dot{s}(t) \geq 0$$
$$\bar{\tau}(s(t)) \geq \tau(t) \geq \underline{\tau}(s(t))$$

Time-optimal path tracking problem[1] (function in s)

$$\underset{a(\cdot), b(\cdot), \tau(\cdot)}{\text{minimize}} \int_0^1 b(s)^{-1/2} ds$$

$$\tau(s) = m(s)a(s) + c(s)b(s) + g(s)$$
$$s \in [0, 1]$$
$$b(0) = \dot{s}_0^2$$
$$b(1) = \dot{s}_T^2$$
$$b'(s) = 2a(s)$$
$$b(s) \geq 0$$
$$\bar{\tau}(s) \geq \tau(s) \geq \underline{\tau}(s)$$

[1] Diederik Verscheure et al. "Time-optimal path tracking for robots: A convex optimization approach". In: *IEEE Transactions on Automatic Control* 54.10 (2009), pp. 2318–2327

Something relevant to you?

Time-optimal path tracking problem[1] (functionals in s)

$$\underset{a(\cdot),b(\cdot),\tau(\cdot)}{\text{minimize}} \int_0^1 b(s)^{-1/2} ds$$

$$\tau(s) = m(s)a(s) + c(s)b(s) + g(s)$$
$$s \in [0, 1]$$
$$b(0) = \dot{s}_0^2$$
$$b(1) = \dot{s}_T^2$$
$$b'(s) = 2a(s)$$
$$b(s) \geq 0$$
$$\bar{\tau}(s) \geq \tau(s) \geq \underline{\tau}(s)$$

PhD course in Fall, 2023

- Optimal control formulation
  → convex or non-convex
- Numerical methods
  → NLP (finite dimension)

Study circle in convex optim.

- NLP (convex)

Key points

Reformulation of problem in path coordinate s and change of variables to $a(s), b(s)$.

[1] Diederik Verscheure et al. "Time-optimal path tracking for robots: A convex optimization approach". In: *IEEE Transactions on Automatic Control* 54.10 (2009), pp. 2318–2327

A two-link example: optimistic about the reality

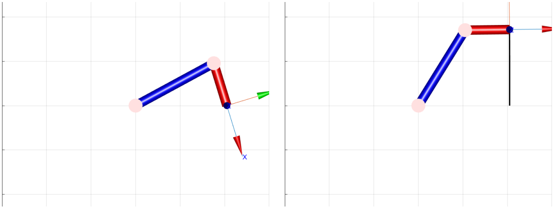
Examples

The mass of each link, m_i^{model}, of the nominal model used in the optimization is 5 percent **smaller** than the actual m_i^{robot}:

$$0.95 m_i^{\mathrm{robot}} = m_i^{\mathrm{model}} < m_i^{\mathrm{robot}} \tag{10}$$

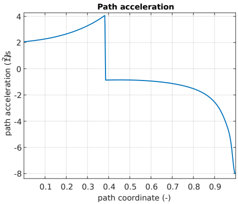
which implies

$$T^{model} < T^{robot} \tag{11}$$

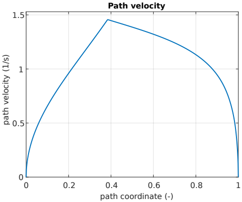


(a) Initial pose (b) Final pose and path

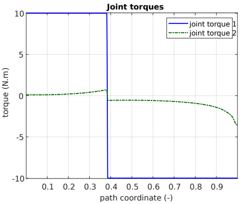
A two-link example: optimistic optimal solutions



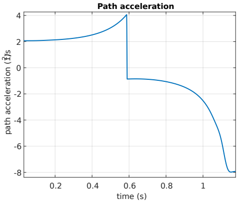
(a) $a(s) = \ddot{s}$



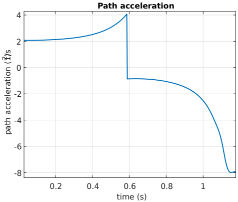
(b) $\sqrt{b(s)} = \dot{s}$



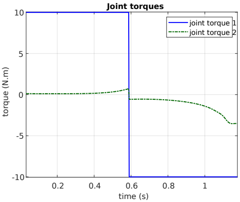
(c) $\tau(s)$



(d) $a(s(t)) = \ddot{s}$



(e) $\sqrt{b(s(t))} = \dot{s}$



(f) $\tau(s(t))$

Figure 3: Optimistic solutions. Upper: in path coordi. in s. Lower: in time (sec).

Path deviation

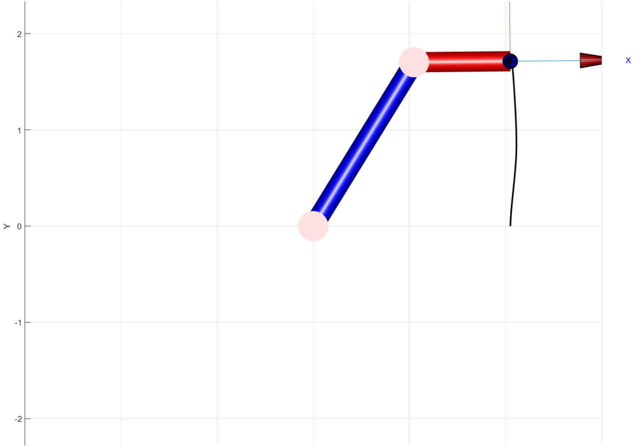
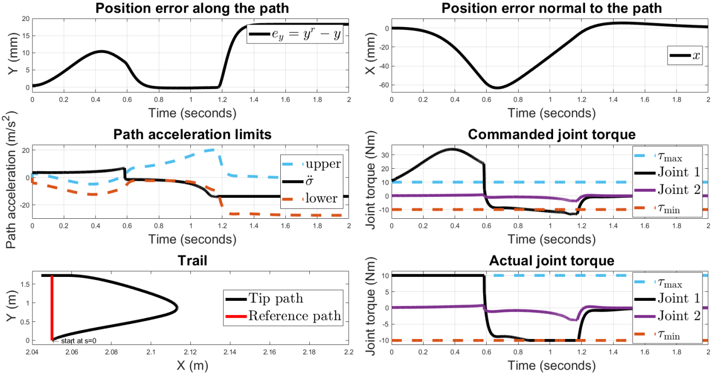


Figure 4: Tip path

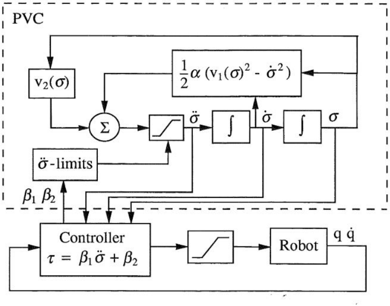
A two-link example: tracking optimistic solutions

- Reference optimal solutions are at the torque limits.
- Modeling error introduce tracking error.
- Controller commands torques beyond the torque limits → control performance is lost.
- Possible solution: slow down the path velocity $\sqrt{b(s)}$

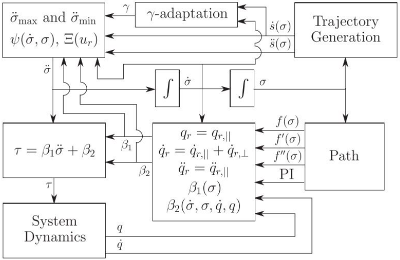


Path velocity control

Seminal work by Ola Dahl and Lars Nielsen. Further investigation by Björn Olofsson and Lars Nielsen.



(a) Path velocity control[1]



(b) Path tracking velocity control[2]

Note that $\dot{\sigma}$ means adjusted path velocity. $\dot{\sigma} \rightarrow v_1(\sigma) = \sqrt{b(\sigma)}$

[1] Ola Dahl. "Path Constrained Robot Control". PhD thesis. Department of Automatic Control, 1992

[2] Björn Olofsson and Lars Nielsen. "Path-tracking velocity control for robot manipulators with actuator constraints". In: *Mechatronics* 45 (2017), pp. 82–99

Path velocity control

Seminal work by Ola Dahl and Lars Nielsen. Further investigation by Björn Olofsson and Lars Nielsen.

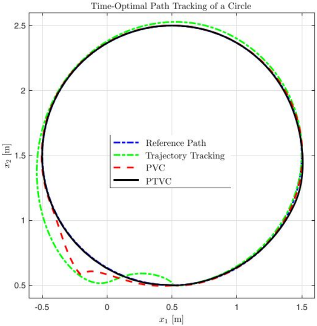


Figure 6: Comparison of controllers[2]

Path velocity control

- Tangential control: joint space

Path-tracking velocity control

- Tangential control: joint space
- Transversal control: Cartesian space

[1] Ola Dahl. "Path Constrained Robot Control". PhD thesis. Department of Automatic Control, 1992

[2] Björn Olofsson and Lars Nielsen. "Path-tracking velocity control for robot manipulators with actuator constraints". In: *Mechatronics* 45 (2017), pp. 82–99

Concept of Cartesian impedance control

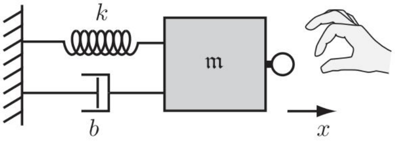


Figure 7: A robot creates a virtual one-dof mass-spring-damper system [1]

Reasons to look at Cartesian impedance control:

- Safe force control + path tracking

[1] Kevin M Lynch and Frank C Park. *Modern robotics*. Cambridge University Press, 2017

[2] Neville Hogan. "Impedance Control: An Approach to Manipulation: Part I—III". In: *Journal of Dynamic Systems, Measurement, and Control* 107.1 (Mar. 1985), pp. 1–24

[3] Christian Ott. *Cartesian impedance control of redundant and flexible-joint robots*. Springer, 2008

Concept of Cartesian impedance control

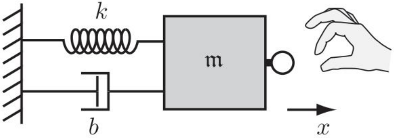


Figure 7: A robot creates a virtual one-dof mass-spring-damper system [1]

Reasons to look at Cartesian impedance control:

- Safe force control + path tracking
- Similar to PTVC, it regulates error in Cartesian space

[1] Kevin M Lynch and Frank C Park. *Modern robotics*. Cambridge University Press, 2017

[2] Neville Hogan. "Impedance Control: An Approach to Manipulation: Part I—III". In: *Journal of Dynamic Systems, Measurement, and Control* 107.1 (Mar. 1985), pp. 1–24

[3] Christian Ott. *Cartesian impedance control of redundant and flexible-joint robots*. Springer, 2008

Concept of Cartesian impedance control

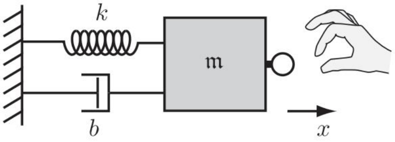


Figure 7: A robot creates a virtual one-dof mass-spring-damper system [1]

Reasons to look at Cartesian impedance control:

- Safe force control + path tracking
- Similar to PTVC, it regulates error in Cartesian space
- It can be parameterized by path velocity control

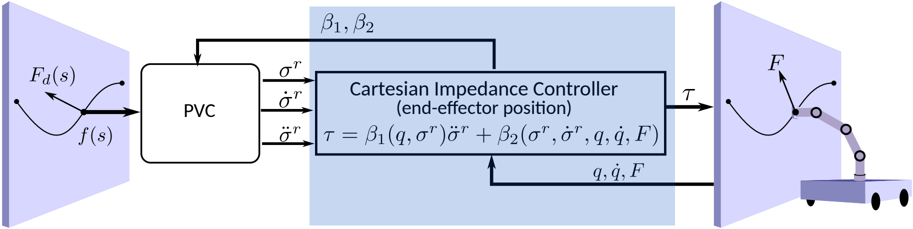
[1] Kevin M Lynch and Frank C Park. *Modern robotics*. Cambridge University Press, 2017

[2] Neville Hogan. "Impedance Control: An Approach to Manipulation: Part I—III". In: *Journal of Dynamic Systems, Measurement, and Control* 107.1 (Mar. 1985), pp. 1–24

[3] Christian Ott. *Cartesian impedance control of redundant and flexible-joint robots*. Springer, 2008

Table of Contents

Mobile manipulator + Path tracking + Force interaction



- Path tracking & force interaction → Cartesian impedance control[1]
- Modeling uncertainty & time optimal → Path velocity control (PVC)[2,3]
  1. Time-optimal trajectories → some joint torque(s) which is/are saturated (not necessarily all, and these may furthermore be artificially set lower on purpose than the physical limits).
  2. Modeling uncertainty → additional joint torques beyond the limit

[1] Hogan, "Impedance Control: An Approach to Manipulation: Part I—III".

[2] Ola Dahl. "Path Constrained Robot Control". PhD thesis. Department of Automatic Control, 1992.

[3] Björn Olofsson and Lars Nielsen. "Path-tracking velocity control for robot manipulators with actuator constraints". In: *Mechatronics* 45 (2017), pp. 82–99.

Cartesian impedance control + Path velocity control

Path parameterization of Cartesian impedance control

Joint torques using Cartesian impedance control can be parameterized by references $\sigma^r, \dot{\sigma}^r, \ddot{\sigma}^r$, states $q, \dot{q}$, and external force F.

$$\tau(\ddot{\sigma}^r) = \beta_1(q, \sigma^r)\ddot{\sigma}^r + \beta_2(\sigma^r, \dot{\sigma}^r, q, \dot{q}, F) \tag{12}$$

Cartesian impedance control + Path velocity control

Path parameterization of Cartesian impedance control

Joint torques using Cartesian impedance control can be parameterized by references $\sigma^r, \dot{\sigma}^r, \ddot{\sigma}^r$, states $q, \dot{q}$, and external force F.

$$\tau(\ddot{\sigma}^r) = \beta_1(q, \sigma^r)\ddot{\sigma}^r + \beta_2(\sigma^r, \dot{\sigma}^r, q, \dot{q}, F) \tag{12}$$

Path acceleration limits

Given joint torque limits $\tau_{\min} \leq \tau(\ddot{\sigma}^r) \leq \tau_{\max}$,

$$\ddot{\sigma}_{\min}(\beta_1, \beta_2) \leq \ddot{\sigma}^r \leq \ddot{\sigma}_{\max}(\beta_1, \beta_2) \tag{13}$$

Cartesian impedance control + Path velocity control

Path parameterization of Cartesian impedance control

Joint torques using Cartesian impedance control can be parameterized by references $\sigma^r, \dot{\sigma}^r, \ddot{\sigma}^r$, states $q, \dot{q}$, and external force F.

$$\tau(\ddot{\sigma}^r) = \beta_1(q, \sigma^r)\ddot{\sigma}^r + \beta_2(\sigma^r, \dot{\sigma}^r, q, \dot{q}, F) \tag{12}$$

Path acceleration limits

Given joint torque limits $\tau_{\min} \leq \tau(\ddot{\sigma}^r) \leq \tau_{\max}$,

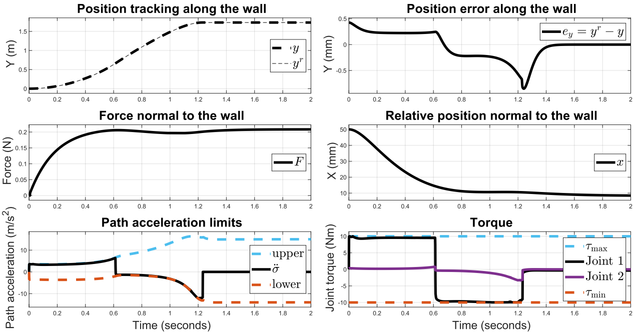
$$\ddot{\sigma}_{\min}(\beta_1, \beta_2) \leq \ddot{\sigma}^r \leq \ddot{\sigma}_{\max}(\beta_1, \beta_2) \tag{13}$$

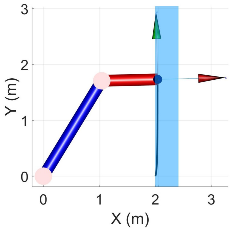
Path velocity control (PVC)

When $q \neq q^r, \dot{q} \neq \dot{q}^r, F \neq 0$, $\tau_{\min} \leq \tau(\ddot{\sigma}^r) \leq \tau_{\max}$ may not hold for $\ddot{\sigma}^r$. Limit the path acceleration $\ddot{\sigma}$ **instantaneously** such that

$$\tau_{\min} \leq \tau(\ddot{\sigma}) \leq \tau_{\max} \tag{14}$$

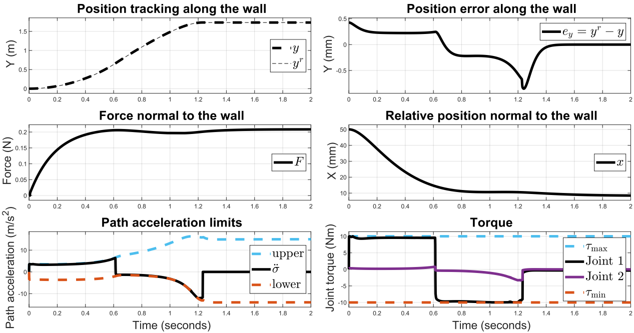
Simulation: end-effector from bottom to top

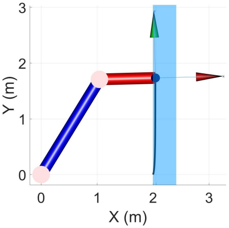




$$m_i^{\text{model}} = 0.95 m_i^{\text{robot}}$$
$$K_{\text{wall}} = 5 \text{ N/m}$$
$$D_{\text{wall}} = 1 \text{ Ns/m}$$

- PVC limits the acceleration from $t \approx 0.0$ s to $t \approx 1.2$ s





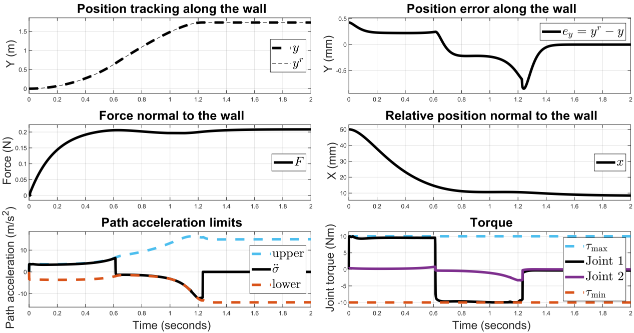
$$m_i^{\text{model}} = 0.95 m_i^{\text{robot}}$$

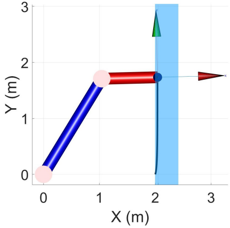
$$K_{\text{wall}} = 5 \text{ N/m}$$

$$D_{\text{wall}} = 1 \text{ Ns/m}$$

- PVC limits the acceleration from $t \approx 0.0$ s to $t \approx 1.2$ s
- Impedance control establishes constant contact force $F \approx 0.2$ N

Simulation: end-effector from bottom to top

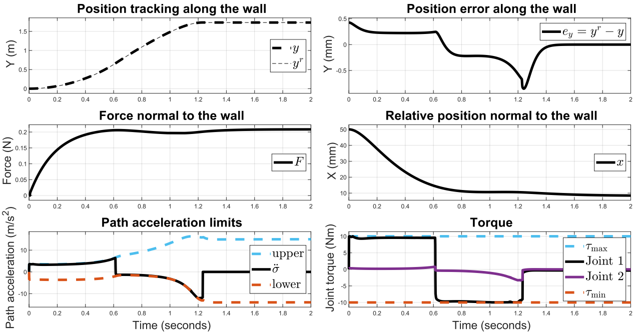


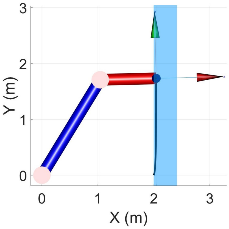


$$m_i^{\text{model}} = 0.95 m_i^{\text{robot}}$$
$$K_{\text{wall}} = 5 \text{ N/m}$$
$$D_{\text{wall}} = 1 \text{ Ns/m}$$

- PVC limits the acceleration from $t \approx 0.0$ s to $t \approx 1.2$ s
- Impedance control establishes constant contact force $F \approx 0.2$ N
- Impedance control tracks the reference path with error $|e_y| \approx 0.7$ mm

Simulation: end-effector from bottom to top





$$m_i^{\text{model}} = 0.95 m_i^{\text{robot}}$$
$$K_{\text{wall}} = 5 \text{ N/m}$$
$$D_{\text{wall}} = 1 \text{ Ns/m}$$

- PVC limits the acceleration from $t \approx 0.0$ s to $t \approx 1.2$ s
- Impedance control establishes constant contact force $F \approx 0.2$ N
- Impedance control tracks the reference path with error $|e_y| \approx 0.7$ mm
- Joint torques are kept within the limits -10 Nm $\leq \tau \leq$ 10 Nm

Table of Contents

Inadmissible acceleration limits

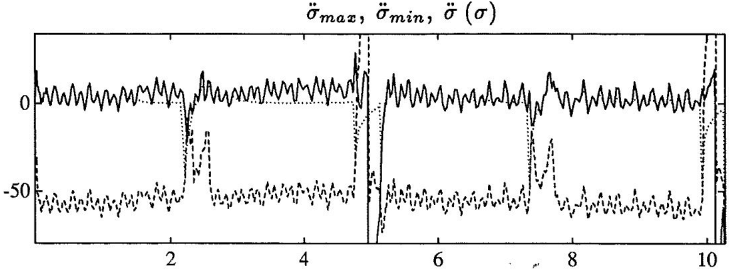
Path acceleration limits

$\ddot{\sigma}_{\min}(\beta_1, \beta_2) \leq \ddot{\sigma}^r \leq \ddot{\sigma}_{\max}(\beta_1, \beta_2)$

Since path acceleration limits depend on measured $q, \dot{q}, F$, and references $\sigma^r, \dot{\sigma}^r$, inadmissible acceleration limits may occur.

Inadmissible acceleration limits

$\ddot{\sigma}_{\min}(\beta_1, \beta_2) > \ddot{\sigma}_{\max}(\beta_1, \beta_2)$



Predict inadmissible acceleration limits

Compute a sequence of $\ddot{\sigma}_{1:K}$ such that

$$\ddot{\sigma}_{\min}(\beta_{1,k}, \beta_{2,k}) \leq \ddot{\sigma}_k \leq \ddot{\sigma}_{\max}(\beta_{1,k}, \beta_{2,k}), \quad k = 1, \dots, K \tag{15}$$

Or equivalently,

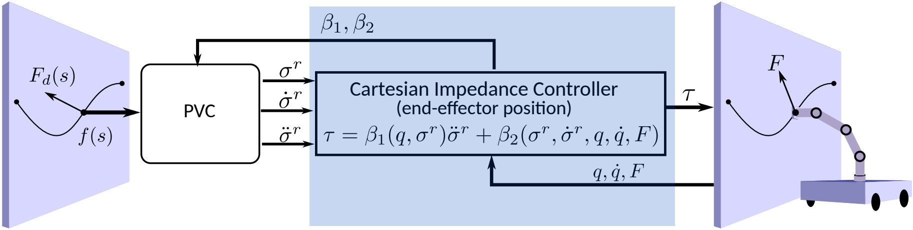
$$\tau_{\min} \leq \tau(\ddot{\sigma}_k) \leq \tau_{\max}, \quad k = 1, \dots, K \tag{16}$$

$$\begin{aligned}
\underset{u_{\ddot{\sigma}^r}(t)}{\text{minimize}} \quad & \int_0^T L\left(\xi(t), u_{\ddot{\sigma}^r}(t)\right) dt \\
\text{subject to} \quad & \dot{\xi}(t) = f(\xi(t), u_{\ddot{\sigma}^r}(t)) \\
& \tau(t) = \tau(\xi(t)) \\
& \xi(0) = \text{measured current states} \\
& \dot{\sigma}^r(t) \geq 0 \\
& \sigma_f \geq \sigma^r(t) \geq \sigma_0 \\
& \bar{\tau} \geq \tau(\xi(t)) \geq \underline{\tau}
\end{aligned}
\qquad \xi(t) = \begin{pmatrix} \sigma^r \\ \dot{\sigma}^r \\ q \\ \dot{q} \\ F \end{pmatrix}(t)$$

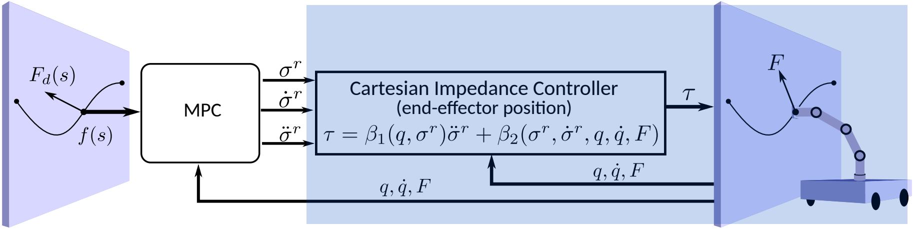
Figure 8: Predict "closed-loop" system behavior

Predictive velocity control + Cartesian impedance control

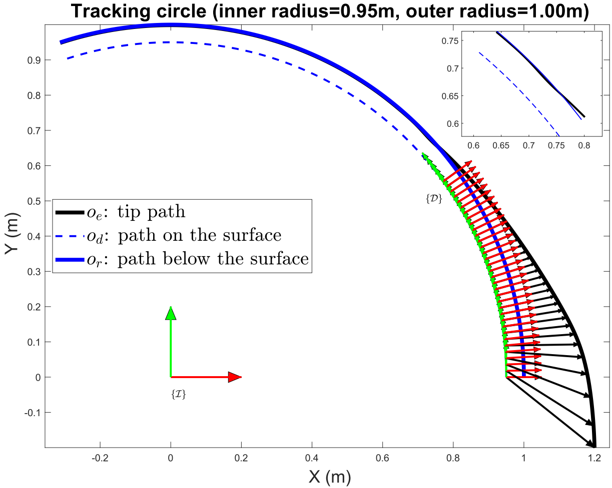
- Instantaneously limit acceleration



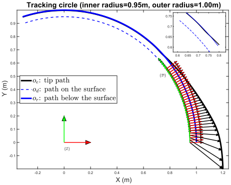
- Predict and limit acceleration



Generalization to curves for 2D and surfaces for 3D



Generalization to curves for 2D and surfaces for 3D



Error dynamics in rotating frame

$$ {}_{\mathcal{D}}e = R_{\mathcal{I}\mathcal{D}}^{T} \, {}_{\mathcal{I}}e \tag{17} $$

$$ {}_{\mathcal{D}}\dot{e} = \dot{R}_{\mathcal{I}\mathcal{D}}^{T} \, {}_{\mathcal{I}}e + R_{\mathcal{I}\mathcal{S}}^{T} \, {}_{\mathcal{I}}\dot{e} \tag{18} $$

$$ {}_{\mathcal{D}}\ddot{e} = \ddot{R}_{\mathcal{I}\mathcal{D}}^{T} \, {}_{\mathcal{I}}e + \dot{R}_{\mathcal{I}\mathcal{D}}^{T} \, {}_{\mathcal{I}}\dot{e} + \dot{R}_{\mathcal{I}\mathcal{D}}^{T} \, {}_{\mathcal{I}}\dot{e} + R_{\mathcal{I}\mathcal{D}}^{T} \, {}_{\mathcal{I}}\ddot{e} \tag{19} $$

Cartesian impedance relationship

$$ M_v \, {}_{\mathcal{D}}\ddot{e} + D_v \, {}_{\mathcal{D}}\dot{e} + K_v \, {}_{\mathcal{D}}e = {}_{\mathcal{D}}f_{ext} \tag{20} $$

$$ K_v = \begin{bmatrix} 750 & 0 \\ 0 & 30 \end{bmatrix} \tag{21} $$

Joint torque

$$ \tau(t) = \tau({}_{\mathcal{I}}\ddot{e}) \tag{22} $$

Summary

Welcome collaboration and any feedback

- Time-optimal solutions need to be treated carefully
- Possible to combine velocity control (instantaneous or predictive) and impedance control to achieve decoupled path-tracking and force control.