LUND UNIVERSITY

Gourmet Dinner of Complex-Coefficient Systems

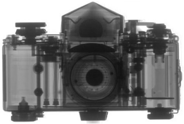
— The Second Serving

Olof Troeng 2020-10-02

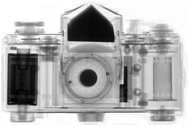
Outline

- Motivation: Cavity Field Control for ESS
- Control for Complex-Coefficient Systems
- More recent uses
  - Intuitive tuning of disturbance rejecting peak filters
  - Analyzing an academically interesting optimization problem using μ
  - Understanding low-latency digital downconversion
  - Widely linear systems

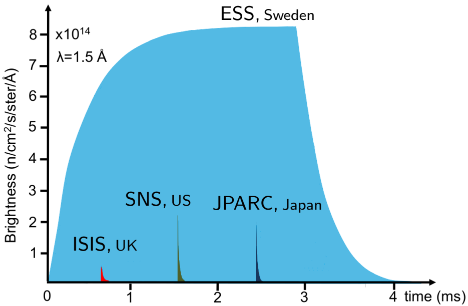
Neutrons Reveal "Invisible" Features



X-Rays

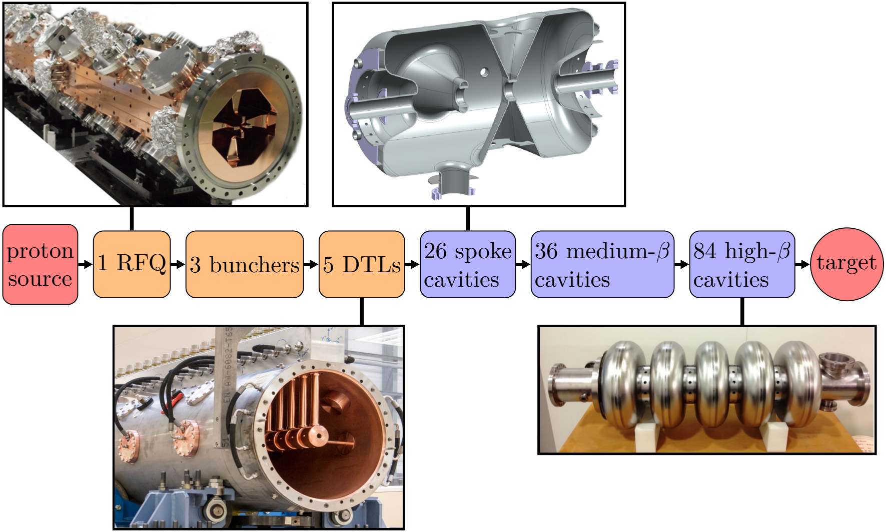


Neutrons

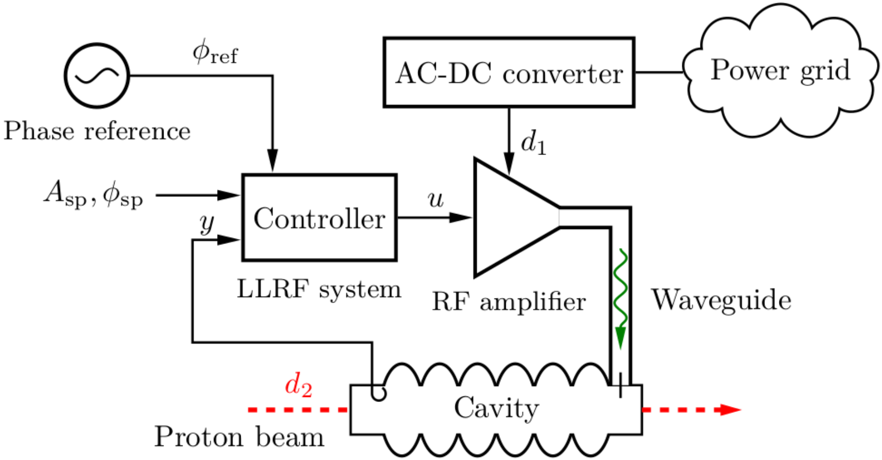


- The Europan Spallation Source is being built outside of Lund
- The world's brightest neutron source
- ...driven by the world's most powerful linear accelerator
- 2B€ European Collaboration

The ESS Accelerator



The Field Control Loop



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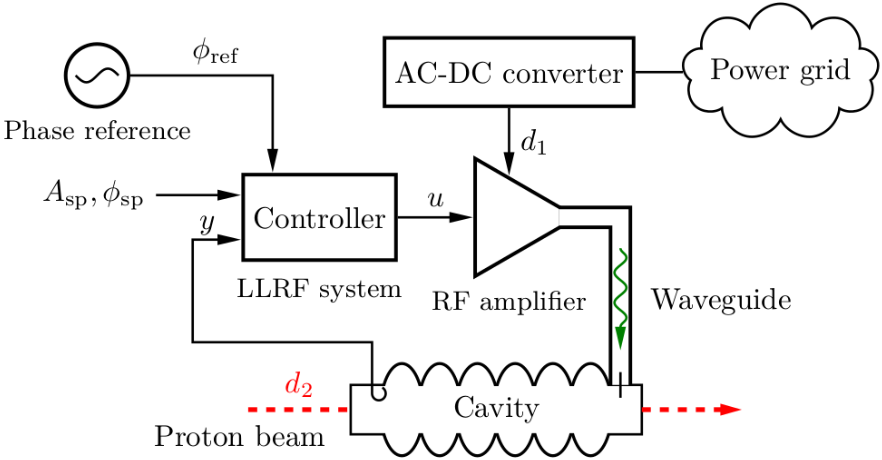
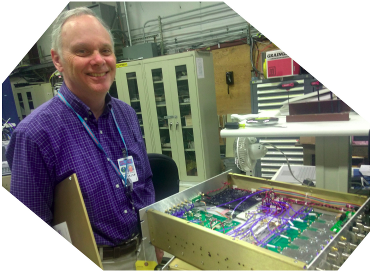


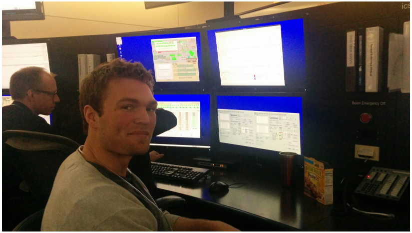
Objective: keep amplitude and phase of y at set points, otherwise

Lab Visits



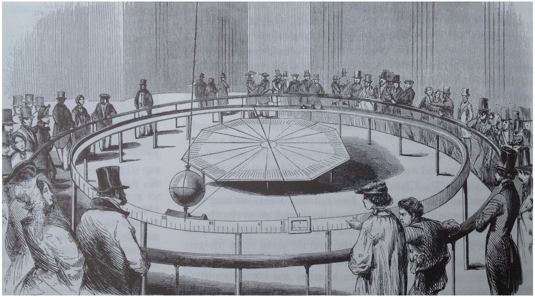
Berkeley Lab, CA

European XFEL, Hamburg



SNS, Knoxville, TN

Origins 1: Rotational invariance



Differential equations for the Foucault pendulum:

$$\ddot{x} = -\frac{g}{l}x + 2\omega\dot{y}\sin\lambda$$

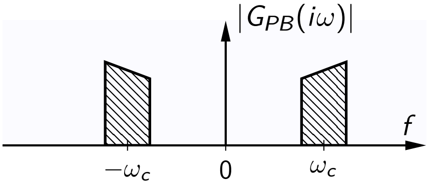
$$\ddot{y} = -\frac{g}{l}y - 2\omega\dot{x}\sin\lambda$$

By introducing $z = x + iy$, we can write:

$$\ddot{z} = -\frac{g}{l}z - 2i\omega\dot{z}\sin\lambda$$

Origins 2: Baseband transformation

Consider passband systems $G_{PB}(s)$, with narrow support around ω_c



$$u(t) = A(t)\cos\Big(\omega_c t + \phi(t)\Big) \in \mathbb{R}$$

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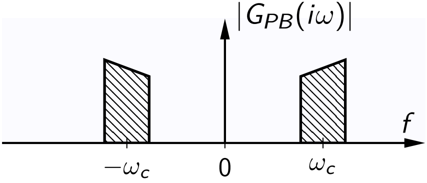
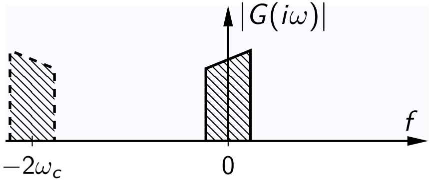


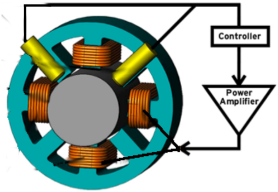
$$u(t) = A(t)\cos\Big(\omega_c t + \phi(t)\Big) \in \mathbb{R}$$

Baseband transformation $s \mapsto s - i\omega_c$, gives $G(s) = G_{PB}(s + i\omega_c)$



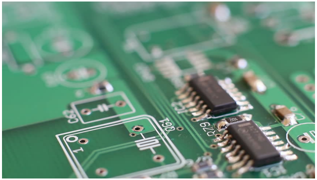
$$u(t) = A(t)\mathrm{e}^{i\phi(t)} \in \mathbb{C}$$

Important Applications

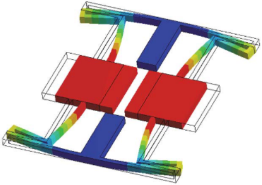


Magnetic bearings

Power electronics

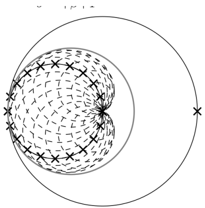


RF amplifier feedback linearization (Sec. 4.6.2)



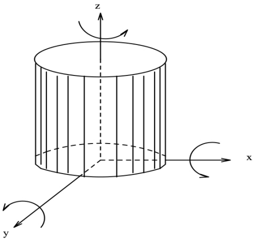
MEMS Gyroscopes

Other Applications



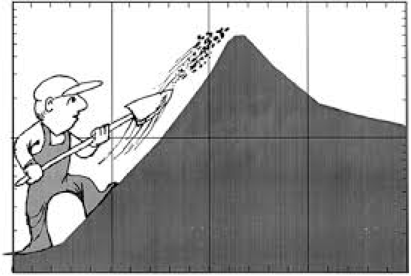
Contraction factor of operators for splitting methods

Ball-on-plate (trivial)



Doyle's spinning satellite

Bode's Sensitivity Integral



Bode's sensitivity integral:

$$\int_0^\infty \log |S(i\omega)| \, d\omega = \pi \sum_{k=1}^{N_p} \mathrm{Re}\, p_k$$

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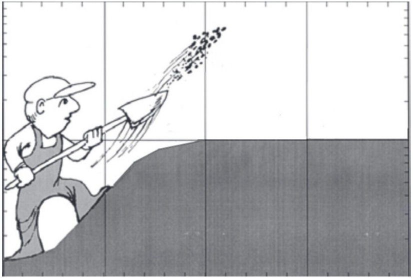

Bode's sensitivity integral:

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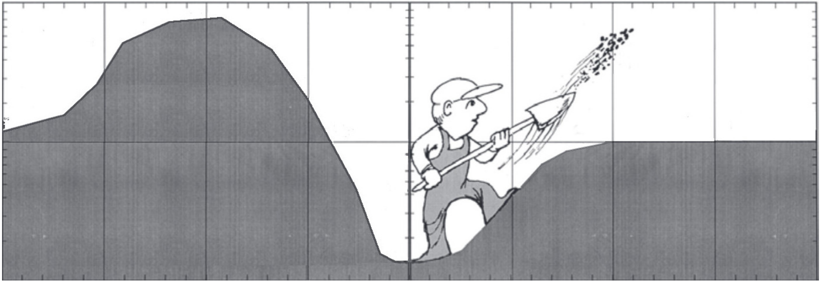
Does not hold!

Bode's Sensitivity Integral



Bode's sensitivity integral:

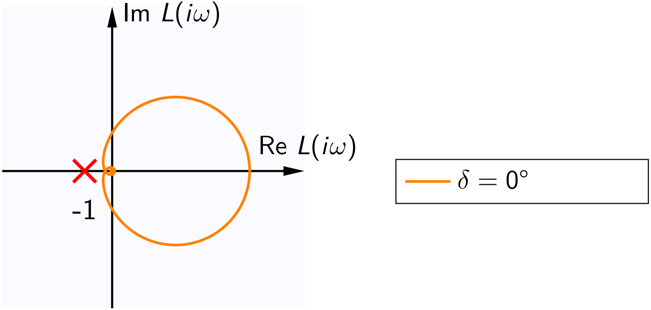
$$\int_{\cancel{0}\,-\infty}^{\infty} \log|S(i\omega)|\,d\omega = \mathbf{2}\pi \sum_{k=1}^{N_p} \operatorname{Re} p_k,$$

Loop Phase Adjustment

Open loop transfer function

$$L(s) = P_{\mathsf{cav}}(s)\mathrm{e}^{-sL}\mathrm{e}^{-i\theta} \cdot C_0(s)\mathrm{e}^{i\theta_{\mathsf{adj}}} = L_0(s)\mathrm{e}^{i\delta}$$

Stability and robustness depends on loop phase adjustment error $\delta = \theta_{\mathsf{adj}} - \theta$



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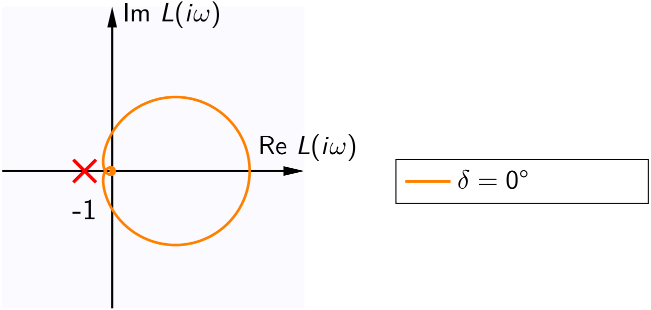

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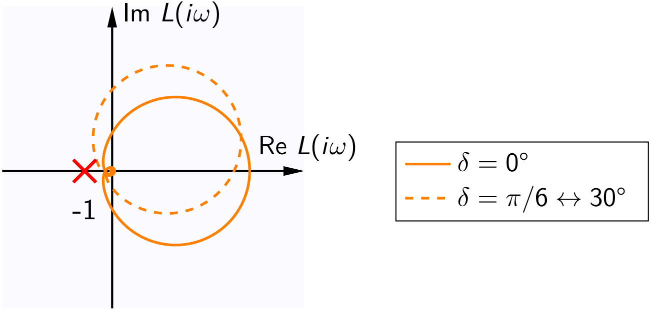

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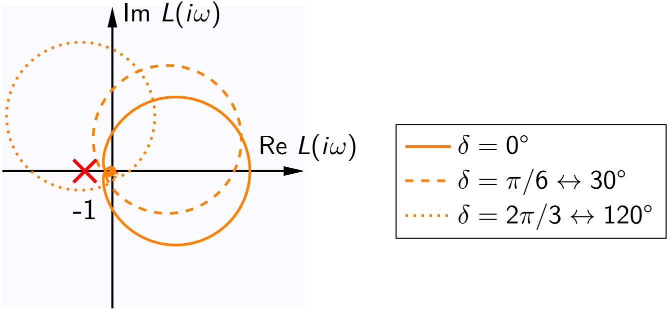
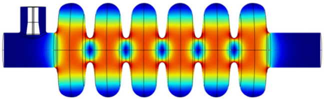
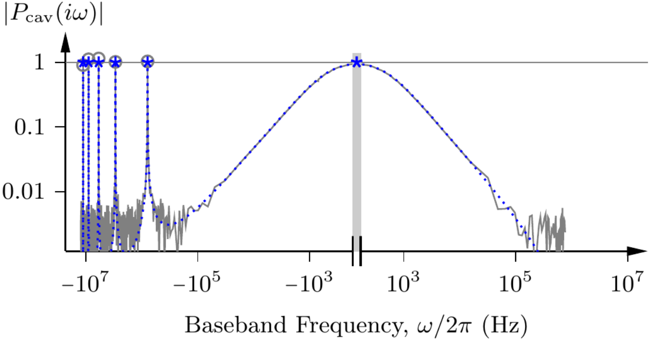


Modeling of Parasitic Cavity Modes



$$P_{\mathrm{cav}}(s) = \frac{\gamma}{s + \gamma - i\Delta\omega}$$



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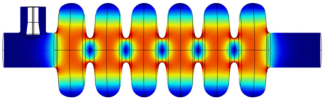
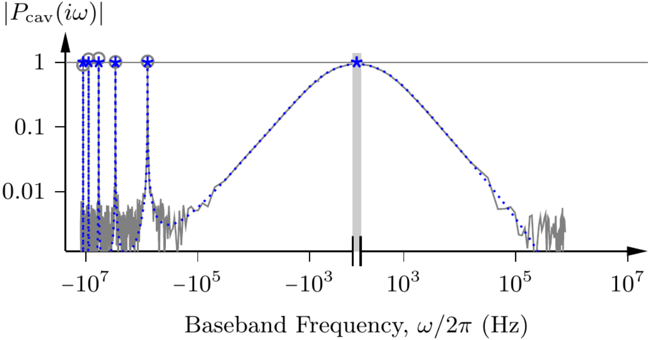
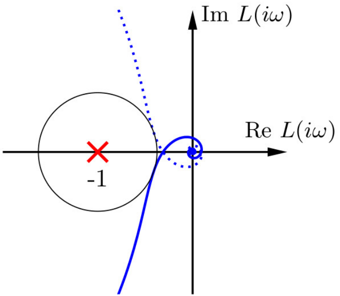


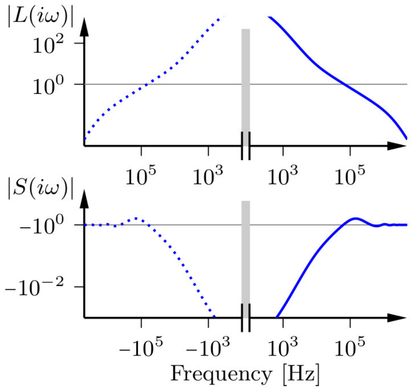
$$P_{\mathrm{cav}}(s) = \frac{\gamma_\pi}{s + \gamma_\pi - i\Delta\omega_\pi} + \gamma_\pi \sum_{n=2}^{N} (-1)^{N-n} \frac{R_n^2}{s + \gamma_n - i\Delta\omega_n}$$



Parasitic modes

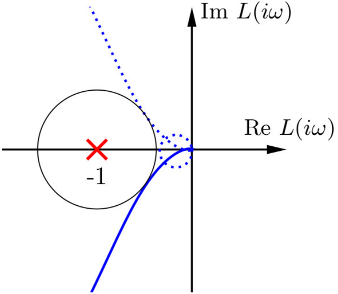
Nominal design, no parasitic modes, PI + 1st order filter:

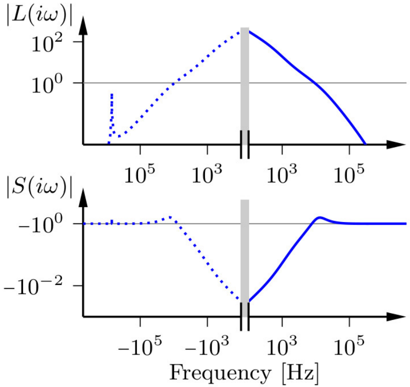




Parasitic modes, control strategies (1/4)

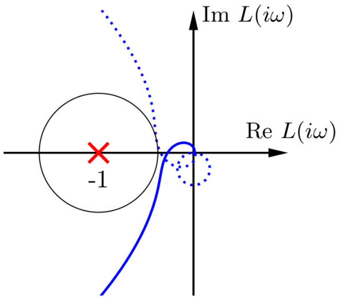
PI + 1st order filter:

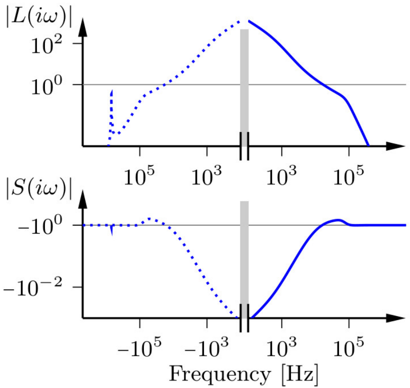




Parasitic modes, control strategies (2/4)

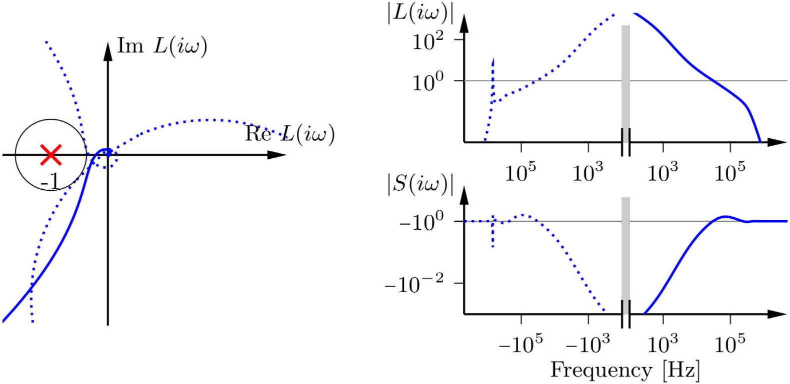
PI + 2nd order filter:



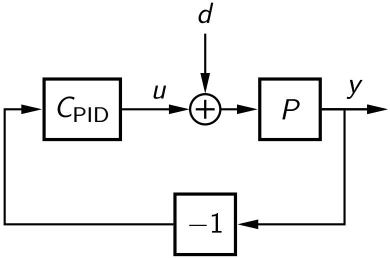


Parasitic modes, control strategies (4/4)

PI + 3rd order filter, adjusting phase of resonant "bulge":



Problem: Narrowband Disturbances



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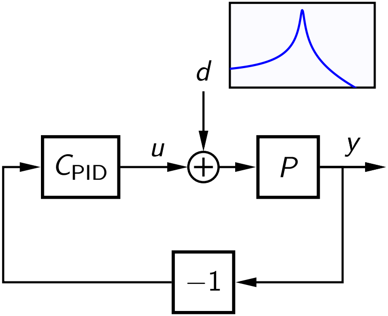

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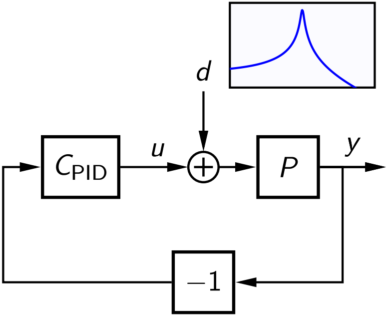

PID-controller is too simplistic: "No internal disturbance model"

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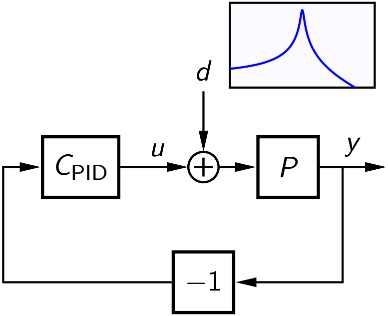

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How to keep PID structure <u>and</u> reject narrowband disturbances?

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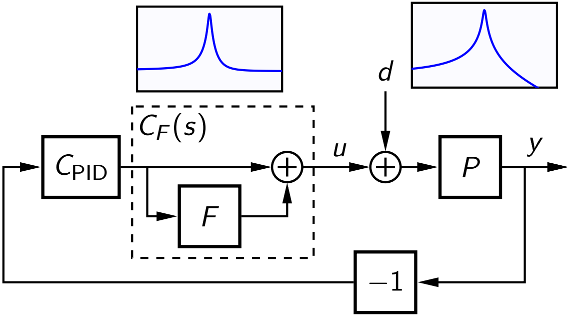

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How to keep PID structure and reject narrowband disturbances?
Increase controller gain at disturbance frequency using peak-filter

$$C_F(s) = \frac{s^2 + 2\zeta_z\omega_z s + \omega_z^2}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2}, \qquad \zeta_z > \zeta_0, \quad \omega_z \approx \omega_0$$

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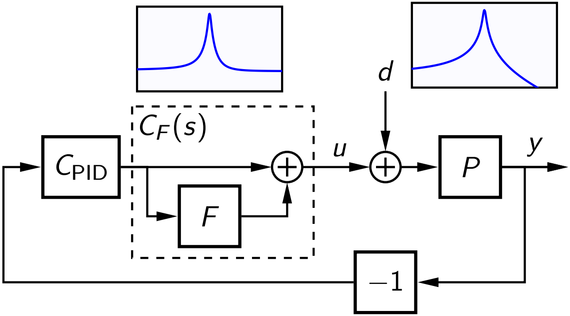

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This talk: Intuitive Method for selecting the filter parameters

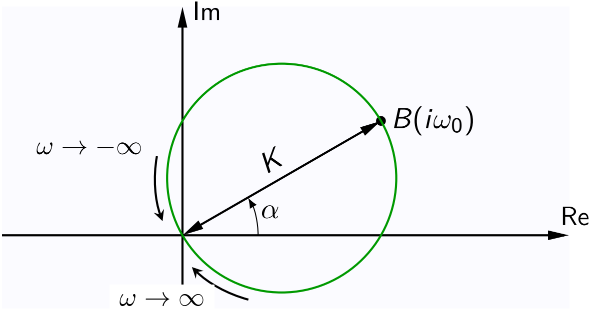
Proposed Filter Parametrization

$$F(s) := K\frac{2\zeta_0\omega_0(s\cos\alpha - \omega_0\sin\alpha)}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2} \approx B(s) + B^*(s)$$

where

$$B(s) = K\mathrm{e}^{i\alpha}\frac{\zeta_0\omega_0}{s - i\omega_0 + \zeta_0\omega_0}$$

$B(s)$ is a complex-coefficient filter with circular Nyquist curve



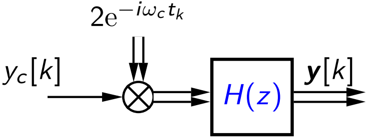
Digital Downconversion

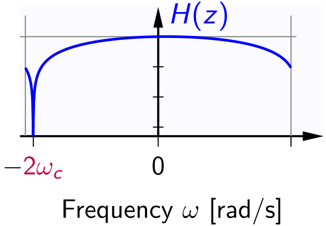
Two-sample reconstruction recovers the complex envelope $\boldsymbol{y}[t_k]$ of a sampled sinusoidal $y_c[t_k] = \mathrm{Re}\{\boldsymbol{y}[t_k]\mathrm{e}^{-i\omega_c t_k}\}$ with low latency.

Traditionally analyzed as:

$$\begin{bmatrix} \boldsymbol{y}_{\mathrm{re}}[k] \\ \boldsymbol{y}_{\mathrm{im}}[k] \end{bmatrix} = \frac{1}{\sin\Delta} \begin{bmatrix} \sin k\Delta & \sin(k-1)\Delta \\ -\cos k\Delta & \cos(k-1)\Delta \end{bmatrix} \begin{bmatrix} y_c[k-1] \\ y_c[k] \end{bmatrix}$$

Actually, just digital downconversion with $H(z) = b(1 + \mathrm{e}^{-2i\Delta} z^{-1})$

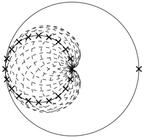


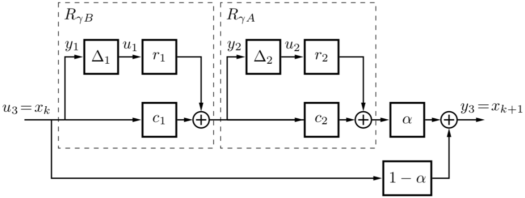


Analyzing Splitting Methods

Composition

- Example with $\sigma = 1$ and $\beta = 1$
- R_A is $\frac{1}{1+\sigma}$ =0.5-negatively averaged, R_B is $\frac{\beta}{1+\beta}$ =0.5-averaged
- Composition $R_B R_A$ is $\frac{\sigma^{-1}+\beta}{\sigma^{-1}+\beta+1}$ =0.67-negatively averaged





Thank you for listening!