Traffic Light Control for Large Scale Networks

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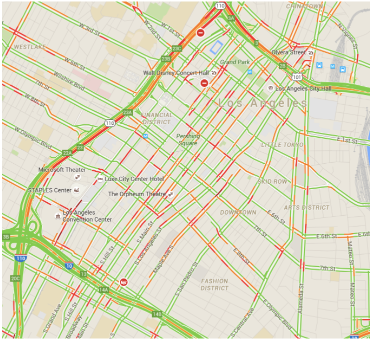


Design traffic light feedback control that is

- Decentralized - Only depend on information nearby
- Scalable - Not depend on the network topology
- Throughput optimality - If possible, the controller should stabilize the network

Previous Work

- Max-pressure controller [Varaiya 2013, Tassiulas & Ephremides 1992]
  - The controllers have explicit information about the turning ratios
- Proportional controller [Savla et. al. 2013, 2014]
  - Acyclic networks
- Queueing networks [Massoulié 2007, Walton 2014]
  - Stochastic setting, fluid approximations have zero equilibrium

Outline

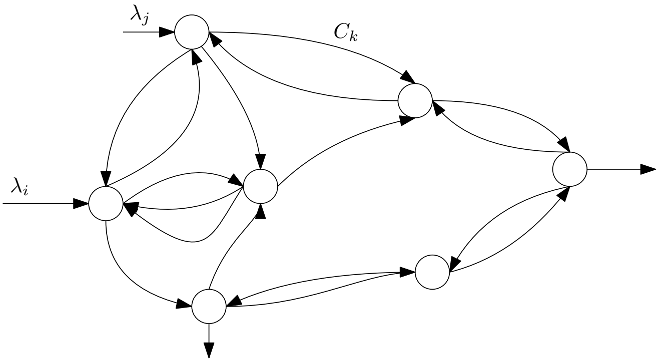
Model

Analysis for the Single Phase

Multiphase Case

Future Work

Model - Network



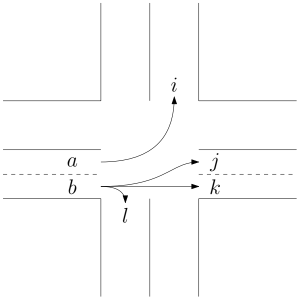
- Capacited multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, C)$.
  - $\mathcal{V}$ - set of intersections
  - $\mathcal{E}$ - set of lanes
  - C - flow capacities
- External inflows λ.

Model - Routing matrix

- R is exogenous.
- R_{ij} - fraction of flow from lane i to lane j
- $R_{ij} > 0 \Rightarrow j$ immediately downstream of i
- $\sum_{j \in \mathcal{E}} R_{ij} \leq 1$, where $(1 - \sum_{j \in \mathcal{E}} R_{ij})$ is the fraction of the flow that will leave the network.

Example

- Lane a: devoted to left turns, $R_{ai} = 1$.
- Lane b: both right turns and straight forward, $R_{bl} = 0.1$, $R_{bj} = 0.3, R_{bk} = 0.6$.

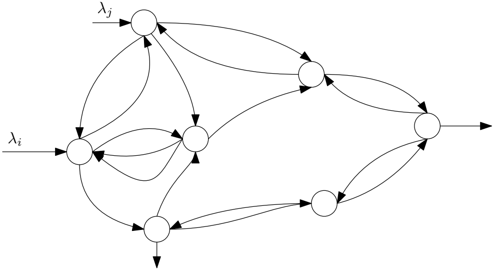


Model - Routing matrix

Assumption

(i) *All lanes can be reached by external inflow, i.e., for every* $i \in \mathcal{E}$ *there exists* $h \in \mathcal{E}$ *such that* $\lambda_h > 0$ *and* $(R^l)_{hi} > 0$ *for some* $l \geq 0$.

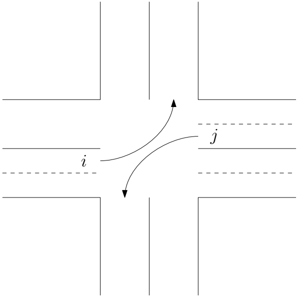
(ii) *It is possible to reach an exit from all lanes, i.e., for every* $i \in \mathcal{E}$ *there exists* $k \in \mathcal{E}$ *such that* $\sum_{j \in \mathcal{E}} R_{kj} < 1$ *and* $(R^l)_{ik} > 0$ *for some* $l \geq 0$.



Model - Phases

For each junction v, introduce a set of phases Ψ_v

- Set of binary vectors $p \in \{0,1\}^{\mathcal{E}_v}$.
- If it possible to activate lane i and j simultaneously, $p_i = p_j = 1$, $p_k = 0$ for all $k \in \mathcal{E}_v \setminus \{i,j\}$.
- Assumed to contain the zero phase, $0 \in \Psi_v$.



The controller's task is to determine the fraction each phase should be activated.

Model - Equilibrium flow

The equilibrium flows on each lane can be computed by

$$a = (I - R^T)^{-1}\lambda.$$

Proposition

A necessary condition for the network to be stabilizable is that, for each $v \in \mathcal{V}$, there exists a vector $\tilde{a}_v$, such that

$$\left(\frac{a_e}{C_e}\right)_{e \in \mathcal{E}_v} \leq \tilde{a}_v \in conv(\Psi_v).$$

Model - Dynamics

- x_i density on lane i
- λ_i external inflow
- C_i the lanes capacity

$$\dot{x}_i = \lambda_i + \sum_{j \in \mathcal{E}} R_{ji} z_j(x) - z_i(x)$$

where $z_i(x) = C_i h_i(x)$ is the outflow from lane i and $1 \geq h_i(x) \geq 0$ determines the amount of green light lane i should receive:

$$h_i(x) = \sum_{p \in \Psi_v} \theta_p^{(v)}(x) p_i,$$

where $\sum_{p \in \Psi_v} \theta_p^{(v)}(x) = 1.$

Model - Maximizing green light policy

$\theta^{(v)}(x^{(v)})$ is determined by concave optimization

$$\theta^{(v)}(x^{(v)}) \in \operatorname*{argmax}_{\theta \in \mathcal{S}_v} \sum_{i \in \mathcal{E}_v} x_i \log \left(\sum_{p \in \Psi_v} \theta_p p_i \right) + \kappa_v \log \theta_0 \,,$$

where $\mathcal{S}_v$ is the simplex of probability vectors over Ψ_v and $\kappa_v > 0$ is the weight on the zero phase.

Analysis Single Phase - Maximizing green light policy

- Every phase can prescribe green light to at most lane
- Set of phases

$$\Psi_v = \{p \in \{0,1\}^{\mathcal{E}_v} : \sum_{e \in \mathcal{E}_v} p_e \leq 1\}$$

- The maximizing green light policy

$$h_i^{(v)}(x^{(v)}) = \frac{x_i}{\sum_{j \in \mathcal{E}_v} x_j + \kappa_v}$$

Analysis Single Phase - Stability

Theorem

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, C)$ be a traffic network topology, R a routing matrix, λ an arrival vector satisfying the previous stated assumptions. Then the dynamical system, with maximizing green light policies, satisfying

$$\sum_{i \in \mathcal{E}_v} \frac{a_i}{C_i} < 1, \quad \forall v \in \mathcal{V}$$

admits a globally asymptotically stable equilibrium ρ^*.

Proof.

Idea: Use the Lyapunov function

$$V(x) = \sum_{i \in \mathcal{E}} x_i \log\left(\frac{z_i(x)}{a_i}\right) + \sum_{v \in \mathcal{V}} \kappa_v \log\left(\frac{h_0^{(v)}(x)}{h_0^{(v)}(x^*)}\right)$$

□

Multiphase Case - Analytical example

Local network, two incoming lanes

$$\dot{x}_1 = 1 - 2h_1(x)$$
$$\dot{x}_2 = 2 - 3h_2(x)$$

One common phase

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} \in \left\langle \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\rangle$$

Explicit solution

$$h_1(x) = h_2(x) = \frac{x_1 + x_2}{x_1 + x_2 + \kappa}$$

Problem: $\lim_{x_1 \to 0} h_1(x) \neq 0$ for all $x_2 > 0$

Possible solution: Differential inclusion

Multiphase Case - Differential inclusion

Introduce the set $\mathcal{I}(t) := \{i \in \mathcal{E} \mid x_i(t) = 0\}$ and $\mathcal{J}(t) := \mathcal{E} \setminus \mathcal{I}$.

- For $j \in \mathcal{J}$:

$$z_j = C_j h_j(x)$$

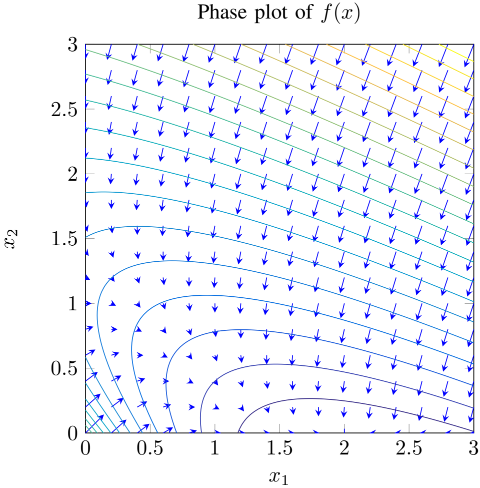
- For $i \in \mathcal{I}$:

$$0 \leq z_j \leq \limsup_{x_i \to 0} C_i h_i(x)$$

$$(I - R'_{\mathcal{I}\mathcal{I}}) z_{\mathcal{I}} \leq \lambda_{\mathcal{I}} + R'_{\mathcal{J}\mathcal{I}} z_{\mathcal{J}}$$

Description of the dynamics change from $\dot{x} = f(x)$ to $\dot{x} \in F(x)$.

Multiphase Case - Phase portrait



Further Work

- Further theoretical investigation of the multiphase case
- Dynamic route choice behavior, i.e., R_{ij} depends on the state of the network
- Finite storage capacities
- Design of different green light policies
- Discrete time analysis
- Apply the controller to the Cell Transmission model/Supply-and-Demand model

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